## Combinational Logic Circuit

## Binary Parallel Adder

Two binary numbers of n bits can be added by using binary parallel adder. A binary parallel adder is a digital function that produces the arithmetic sum of to binary numbers in parallel. It consists of full-adders connected in cascade, with the output carry from one full-adder connect to the input carry of the next full-adder.


Q Design a binary parallel adder to add two 4-bit binary numbers?


Ex Design a BCD to Excess 3 code converter?
Sol
Not Used


Ex Design Excess-3 to BCD code converter?


## BCD adder

It is a combinational circuit that adds two BCD digits in parallel and produces a sum digit also in BCD. Consider the arithmetic addition of two decimal digits in BCD, together with a possible carry from previous stage. Since each input digit does not exceed 9 , the output sum cannot be greater than $9+9+1=19$, where the 1 in the sum being an input carry.

The BCD adder work as

- Suppose two BCD digits are applied to 4 - bit binary adder.
- The adder will form the sum in binary and produce a result in the range 0 to 19 .
- The binary sum is listed in the first column in the table, where $\mathrm{Z}_{1}, \mathrm{Z}_{2}, \mathrm{Z}_{4}, \mathrm{Z}_{8}$ represent the weights $1,2,4,8$ and K is the carry.
- The output sum of two decimal digits must represent in BCD and listed in the second column in the table.
- In second column (BCD sum), where the sum is equal to or less than 1001 , the BCD number is identical, and no conversion is needed.
- When the sum is greater than 1001 . The BCD adder must include the correction logic in its internal construction. The addition of binary $6(0110)$ to binary sum convert it to correct the $B C D$ representation and also produces an output carry.
- From the first column ( Binary Sum ), the correction is needed when the binary sum has an output carry $K=1$, or when 1 in positions $Z_{8}$ and $Z_{4}$, or when 1 in positions $Z_{8}$ and $Z_{2}$.
- The equation is

$$
\mathbf{C}=\mathbf{K}+\mathbf{Z}_{8} \mathbf{Z}_{4}+\mathbf{Z}_{8} \mathbf{Z}_{2}
$$

When the value of $\mathrm{C}=1$ the circuit will add binary $6(0110)$ for correction.

| Binary Sum |  |  |  |  |  | BCD Sum |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| K | $\mathrm{Z}_{8}$ | $\mathrm{Z}_{4}$ | $\mathrm{Z}_{2}$ | $\mathrm{Z}_{1}$ | C | $\mathrm{S}_{8}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{1}$ |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 2 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 3 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 4 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 5 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 6 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 7 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 8 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 9 |
|  |  |  |  |  |  |  |  |  |  |  |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 10 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 11 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 12 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 13 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 14 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 15 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 16 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 17 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 18 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 19 |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |



Block - diagram of a BCD adder

## The Decoder

A decoder is a combinational circuit that converts binary information from n input lines to a maximum of $2^{\mathrm{n}}$ output lines, each output representing one of the minterms of input variables.

Ex Design 3 to 8 decoder?
Sol
The truth table is

| X | Y | Z | D 0 | D 1 | D 2 | D 3 | D 4 | D 5 | D 6 | D 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

3 to 8 line decoder


Ex Design BCD to Decimal decoder?
Sol:
The size of the decoder is $4 \times 16$ and the truth table is

| X | Y | Z | W | D 0 | D 1 | D 2 | D 3 | D 4 | D 5 | D 6 | D 7 | D 8 | D 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

## Don't Care

$\begin{array}{llll}1 & 1 & 0 & 1\end{array}$
$\begin{array}{llll}1 & 1 & 1 & 0\end{array}$

$$
\mathrm{D}_{0}=\mathrm{XYZW} \quad \mathrm{D}_{1}=\mathrm{XYZW}
$$



$$
\mathrm{D}_{2}=\mathrm{YZW} \quad \mathrm{D}_{3}=\mathrm{YZW}
$$

$$
\mathrm{D}_{4}=\mathrm{YZW} \quad \mathrm{D}_{5}=\mathrm{YZW}
$$

$$
\mathrm{D}_{6}=\mathrm{YZW} \quad \mathrm{D}_{7}=\mathrm{XZW}
$$

$$
\mathrm{D}_{8}=\mathrm{XW}
$$

$$
\mathrm{D}_{9}=\mathrm{Y} Z \mathrm{~W}
$$

Ex Implement the Full- Adder circuit with a Decoder and OR gates?

## Sol

From the truth table of a Full-Adder the functions of this combinational circuit in Sum of Minterms
$1-\mathrm{S}(\mathrm{X}, \mathrm{Y}, \mathrm{Z})=\Sigma(1,2,4,7)$
$2-\mathrm{C}(\mathrm{X}, \mathrm{Y}, \mathrm{Z})=\Sigma(3,5,6,7)$
Therefore the number of the inputs in the decoder is $3(\mathrm{X}, \mathrm{Y}, \mathrm{Z})$ and the output is 8 , so the size of the decoder (3x8)


## The Encoder

An encoder is a digital function produces a reverse operation from that of a decoder. An encoder has $2^{\mathrm{n}}$ inputs lines and n output lines generate the binary code for the $2^{\mathrm{n}}$ input variables.

Ex Design the Octal to Binary Encoder
Sol The truth table is

| D0 | D1 | D2 | D3 | D4 | D5 | D6 | D7 | X | Y | Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

D0


Octal to Binary Encoder

## Multiplexer

Multiplexer means transmition a large number of information units over a small number of channels or lines. A digital multiplexer is a combinational circuit that selects binary information one of many inputs lines, there are $2^{\mathrm{n}}$ inputs lines and n selection lines.

Ex $4 \times 1$ multiplexer, the input lines is $4(2 n)$ and the output is one line and the selection lines is 2 .

$4 \times 1$ multiplexer logic circuit

## Boolean Function Implementation

Boolean function can implemented with a multiplexer by following these steps:
1- the variables in the function applied as $n+1$
$2-\mathrm{n}$ is the number of selection lines.
3- The reminder variables taken as input variable
4- The minterms is applied in a table
5- The inputs applied as 0 and 1 as primed and not primed

Ex Implement the following function with a multiplexer.

$$
\mathrm{F}(\mathrm{~A}, \mathrm{~B}, \mathrm{C})=\Sigma(1,3,5,6)
$$

Sol
1- The variables in the function are 3 so $\mathrm{n}+1=3$
2 - The selection lines are $\mathrm{n}=2$ (B,C)
3- The tired variable A is applied as input to the multiplexer.

|  | $\mathrm{I}_{0}$ | $\mathrm{I}_{1}$ | $\mathrm{I}_{2}$ | $\mathrm{I}_{3}$ |
| :--- | :--- | :--- | :--- | :--- |
| A | 0 | 1 | 2 | 3 |
| A | 4 | 5 | 6 | 7 |

4 - The minterms is applied on the table and the block diagram of the multiplexer is drown

H.W.

1- Implement the Boolean function $F=\Sigma(0,1,3,4,8,9,15)$
2 - Implement the following functions using a decoder and OR gates?

$$
\begin{aligned}
& 1-\mathrm{F} 1(\mathrm{~A}, \mathrm{~B})=\mathrm{A}+\mathrm{B} \\
& 2-\mathrm{F} 2(\mathrm{~A}, \mathrm{~B})=\Sigma(0,1) \\
& 3-\mathrm{F} 3(\mathrm{~A}, \mathrm{~B})=\Pi(1,3)
\end{aligned}
$$

## Read Only Memory (ROM )

A read only memory (ROM) is a device that includes both the decoder and OR gates with a single IC package. The ROM is a combinational circuit constructed with a decoder and number of OR gates equal to the number of outputs in the unit. ROM consists of $2^{n} x \mathrm{~m}$, where n is the number of inputs lines and $m$ is the number of outputs lines

## Ex consider 32 x 4 ROM

## Sol

$$
4=\mathrm{m}=\text { outputs lines } \underset{\text { minterm }}{\text { and }} \quad 32=2^{\mathrm{n}}=2^{5} \longrightarrow \mathrm{n}=5=\text { inputs lines }
$$



Logic Construction of $32 \times 4$ ROM

Ex Implement the functions using ROM
$\mathrm{F} 1(\mathrm{~A} 1, \mathrm{~A} 0)=\Sigma(1,2,3)$
$\mathrm{F} 2(\mathrm{~A} 1, \mathrm{~A} 0)=\Sigma(0,2)$

Ex Design a combinational circuit using ROM. The circuit accepts a 3-bit number and generates an output binary number equal to the square of the input numbers.

Sol
The truth table is

| Input |  |  |  |  |  |  |  | Output |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A 2 | A 1 | A 0 | $\mathrm{D}_{5}$ | $\mathrm{D}_{4}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{1}$ | $\mathrm{D}_{0}$ |  |  |  |  |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  |  |  |  |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |  |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |  |  |  |  |  |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |  |  |  |  |  |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |  |  |  |  |  |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |  |  |  |  |  |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |  |  |  |  |  |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

In the output $\mathrm{D}_{0}$ is equal to $\mathrm{A}_{0}$ and D 1 is equal zero therefore the truth table can derived as follow

| Input |  |  |  | Output |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A2 A1 A0 | $\mathrm{D}_{5}$ | $\mathrm{D}_{4}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{2}$ |  |  |  |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |  |  |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 |  |  |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |  |  |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |  |  |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 |  |  |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 |  |  |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 |  |  |

And the block diagram of this ROM


