Review of Number System

Q- Convert the following numbers from a given base to the base indicated?

1- $(1\,1\,0\,1\,0\,1)_2 \longrightarrow (X)_3$

2- $(A77\,.\,C5)_{16} \longrightarrow (X)_2$

3- $(1\,0\,1\,0\,1\,1)_2 \longrightarrow (X)_7$

4- $(6\,A\,.\,2\,0\,5)_{16} \longrightarrow (X)_8$

5- $(1\,2\,3)_4 \longrightarrow (X)_5$

6- $(6\,7\,0\,1\,.\,2\,5\,4)_8 \longrightarrow (X)_{16}$

7- $(325.14)_8 \longrightarrow (X)_{10}$

8- $(1A\,.\,4)_{16} \longrightarrow (X)_{10}$

9- $(67.33)_{10} \longrightarrow (X)_2$

Arithmetic operations in other systems:

عند اجراء العمليات الرياضية الأعتيادية من المحتمل ان ينتج رقم اكبر او يساوي اساس النظام في هذه الحالة يتم قسمة الرقم الناتج على اساس النظام حيث يمثل باقي القسمة ناتج العملية وناتج القسمة هو carry يحمل الى المرتبة الاحقة وفي عملية الطرح تتم الأستدانة من المرتبة الأعلى وقيمة الأستدانة تكون مساوية لأساس النظام المستخدم.

Ex Perform the following operations:-

1 - $(4\,7\,1)_8 + (6\,3\,5)_8 = (1\,3\,2\,6)_8$

2 - $(2A4)_{16} + (CB4)_{16} = (F58)_{16}$

3 - $(4\,0\,5)_8 - (2\,6\,7)_8 = (1\,1\,6)_8$

4 - $(A\,8\,5)_{16} - (5\,D\,4)_{16}$

5 - $(6\,5\,2)_{12} - (4\,8\,0)_{12}$

6 - $(1\,4\,5\,A\,2)_{16} \times (1\,.\,3)_{16}$

7 - $(3\,4\,2)_8 + (12)_{10}$

8 - $(3\,2\,2\,.\,2)_5 - (4\,3\,.\,4)_5$

10- $(5\,3\,7\,.\,4)_{10} + (1\,1\,0\,0\,0\,.\,1\,1)_2$

11- $(5\,A\,4)_{11}\ X\ (2\,.\,3)_{11}$

12- $(1\,0\,0\,1\,1\,.\,0\,1)_2 - (1011\,.\,11)_2$

14- $(1\,A\,B\,.\,8)_{16} - (2\,5\,3\,.\,9)_{10}$

15- $(1\,1\,1\,.\,0\,1)_2\ X\ (1\,.\,0\,1)_2$

Counting in number systems:-

The counting in any system is done by starting with the first digit in the system (0) until the maximum digit of the system is reached and then the counting is continue using 2 digits and so on.

Ex Write first 17 digits in base 8?

Sol:
(0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20)

Ex Write first 30 digits in Hexadecimal system?

Sol:
(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B,1C, 1D)

Ex Write 10 digits in base 8 starting with decimal 5?

Sol:
(5, 6, 7, 10. 11, 12, 13, 14, 15, 16)

The Complement

Complements are used in digital computer for simplifying the subtraction operation and for logical manipulations. There are two types of complements for each base R system:

1– The r s complement.

2 – The (r – 1) s complement.

When the value of the base is substituted, types receive the names 2 s and 1 s complement for binary numbers, or 10 s and 9 s complement for decimal numbers.

1– The r s complement.

The r s comp. of number is obtained by subtracting the first nonzero least significant digit from the base of the system, and subtracting all other higher digits from (base -1).

Ex Find the r s comp. of the following numbers:

1 – $(5\,2\,5\,2\,0)_{10}$ 2 - $(3\,2\,6\,7)_{10}$ 3 - $(2\,5 . 6\,3\,9)_{10}$

4 - $(1\,0\,1\,1\,0\,0)_{2}$ 5 - $(8\,7\,6\,5)_{11}$ 6 - $(A\,0\,9\,0)_{16}$

Sol:

1- The 10 s comp. of $(5\,2\,5\,2\,0)_{10}$ is (47 4 8 0)
2- The 10 s comp. of $(3\,2\,6\,7)_{10}$ is (6 7 3 3)
3- The 10 s comp. of $(2\,5 . 6\,3\,9)_{10}$ is (7 4 . 3 6 1)
4- The 2 s comp. of $(1\,0\,1\,1\,0\,0)_{2}$ is (0 1 0 1 0 0)
5- The 11 s comp. of $(8\,7\,6\,5)_{11}$ is (2 3 4 6)
6 - The 16 s comp. of $(A\,0\,9\,0)_{16}$ is (5 F 7 0)

1– The (r -) s complement.

The (r – 1) s comp. of a number is obtained by subtracting every digit from the (base – 1).

Ex Find the (r -1) s comp. of the following numbers:

1 – $(5\,2\,5\,2\,0)_{10}$ 2 - $(3\,2\,6\,7)_{10}$ 3 - $(2\,5 . 6\,3\,9)_{10}$

4 - $(1\,0\,1\,1\,0\,0)_{2}$ 5 - $(8\,7\,6\,5)_{11}$ 6 - $(A\,0\,9\,0)_{16}$

Sol:

1- The 9 s comp. of $(5\,2\,5\,2\,0)_{10}$ is (47 4 7 9)
2 The 9 s comp. of $(3\,2\,6\,7)_{10}$ is (6 7 3 2)
3 - The 9 s comp. of $(2\,5 . 6\,3\,9)_{10}$ is (7 4 . 3 6 0)
4 - The 1 s comp. of $(1\,0\,1\,1\,0\,0)_{2}$ is (0 1 0 0 1 1)
5 - The 10 s comp. of $(8\,7\,6\,5)_{11}$ is (2 3 4 5)
6 - The 15 s comp. of $(A\,0\,9\,0)_{16}$ is (5 F 6 F)

As a result in ***Binary*** system the 1 s comp. is obtained by change each 1 to 0 and the 0 to 1, and the 2 s comp. is obtained by adding 1 to the 1 s comp.

Ex Find the 1 s and 2 s comp. of $(1\,0\,1\,1\,0 .\ 1\,0\,0)_{2}$?

The negative numbers

There are three types or methods to represent the negative numbers in a computer:-

1- Sign – and – Magnitude (signmagnitude).
2- 1 s complement.
3- 2 s complement.

Ex Represent (+ 12)$_{10}$, (- 12) $_{10}$ in signmagnitude, 1 s and 2 s complement?

Sol :

	Signmagnitude	1 s comp.	2 s comp.
+ 12	0 1 1 0 0	0 1 1 0 0	0 1 1 0 0
- 12	1 1 1 0 0	1 0 0 1 1	1 0 1 0 0

Subtraction with r s complement

The procedure for subtracting two numbers (M – N) with r s comp. is done as follow:

1 – Find the r s comp. of N.
2 – Add M to the result of step 1 .
3– Inspect the result obtained in step 2 for an end carry:-

a – If an end carry occurs, discard it.
b – If an end carry does not occurs, take the r s comp. of the number obtained in step 2 and place negative sign in front it.

Ex Subtract (7 2 5 3 2 - 3 2 5 0) $_{10}$ using 10 s comp.?

Sol:

M = 7 2 5 3 2 , N = 0 3 2 5 0

1 – the 10 s comp. of N is (9 6 7 5 0)

```
2 -          7 2 5 3 2
           + 9 6 7 5 0
            -----------------
       1    6 9 2 8 2
```

3 – The result is (6 9 2 8 2) $_{10}$

Ex Subtract (3250 - 72532)$_{10}$ using rs comp?

Sol :

M=03250 , N=72532

1- The l0 s comp. of N is (2 7 4 6 8)

```
2-        0 3 2 5 0

       +  2 7 4 6 8
    ----------------------------
          3 0 7 1 8          there is no carry
```

3- The l0s comp .of (3 0 7 1 8) is (6 9 2 8 2)

The result is _ (6 9 2 8 2) $_{10}$

Ex Subtract (1 0 0 0 1 0 0) $_2$ _(1 01 0 1 0 0)$_2$ using 2s comp.?

Sol

M=1 0 0 0 1 0 0 , N = 1 0 1 0 1 0 0

1- The 2 s comp of N is (0 1 0 1 1 0 0)

2-
```
    1 0 0  0 1 0 0
 +  0 1 0 1 1 0 0
 ________________
    1 1 1 0 0 0 0
```
there is no carry

3 - The 2s comp . of (1 1 1 0 0 0 0) is (0 0 1 0 0 0 0)

The result - (1 0 0 0 0) $_2$

Subtraction with (r-1) s complement

The procedure for subtracting two numbers (M – N) with (r -1) s comp. is done as follow:

1 – Find the (r-1) s comp. of N.

2- Add M to the result of step 1 .

3– Inspect the result obtained in step 2 for an end carry:-

a – If an end carry occurs, add 1 to the least significant digit.

b – If an end carry does not occurs, take the (r-1) s comp. of the number obtained in step 2 and place negative sign in front it.

Ex Subtract (3250-72532)$_{10}$ using 9 s comp?

Ex Find 83-27 using 1 s comp?

Sol

M = 83 = (1 0 1 0 0 1 1) $_2$, N = 27 = (0 0 1 1 0 1 1) $_2$

1 - The 1 s comp .of N is (1 1 0 0 1 0 0)

2 –
```
      1 0 1 0 0 1 1

 +    1 1 0 0 1 0 0
   ----------------------
1      0 1 1 0 1 1 1
```

3-
```
      0 1 1 0 1 1 1
 +                1
   ------------------
     1 1 1 0 0 0
```
the result is (1 1 1 0 0 0)$_2$

Ex Perform the operation (A B C E F) 16 - (4 8 F 9 D)n16 using (r – 1) s comp.?

Sol:

M = A B C E F , N = 4 8 F 9 D

1 - The 15 s comp. of N is (B 7 0 6 2)

2 -
```
        A  B  C  E  F
   +    B  7  0  6  2
      ------------------
   1    6  2  D  5  1
```

3-
```
        6  2  D  5  1
   +                1
      ----------------
        6  2  D  5  2
```
the result is (6 2 D 5 2) $_{16}$

Binary Codes

Binary codes for decimal digit require a minimum of four bits. Numerous different code can be obtained by arranging four or more bits in many distinct possible combinations. A few possibilities are shown in the table:-

Decimal Digit	(B C D) 8 4 2 1	Excess-3	8 4 -2 -1
0	0 0 0 0	0 0 1 1	0 0 0 0
1	0 0 0 1	0 1 0 0	0 1 1 1
2	0 0 1 0	0 1 0 1	0 1 1 0
3	0 0 1 1	0 1 1 0	0 1 0 1
4	0 1 0 0	0 1 1 1	0 1 0 0
5	0 1 0 1	1 0 0 0	1 0 1 1
6	0 1 1 0	1 0 0 1	1 0 1 0
7	0 1 1 1	1 0 1 0	1 0 0 1
8	1 0 0 0	1 0 1 1	1 0 0 0
9	1 0 0 1	1 1 0 0	1 1 1 1

- In BCD code (Binary Coded Decimal) each decimal number is represented in 4 bits.
- In Excess – 3 Code each decimal number is represented by adding 3 to each number in BCD code.

Ex Convert (13) 10 to Binary , BCD code?

Sol:

- $(13)_{10} = (1\,1\,0\,1)_2$
- $(13)_{10} = (0\,0\,0\,1 \quad 0\,0\,1\,1)_{BCD}$

Ex Decode the following BCD numbers?

1 – $(1\,0\,0\,0\,1\,1\,1\,1\,0\,0\,0\,1\,0\,0\,1\,0\,1\,0\,1)_{BCD}$

2 – $(1\,1\,0\,1\,1\,0\,.\,0\,1\,1\,0\,1)_{BCD}$

Sol :

1 – $(0\,1\,0\,0 \quad 0\,1\,1\,1 \quad 1\,0\,0\,0 \quad 1\,0\,0\,1 \quad 0\,1\,0\,1)_{BCD} = (4\,7\,8\,9\,5)_{10}$

Ex Encode the following numbers into BCD code?

1 – $(1\,1\,1\,0\,0\,1\,1)_2$ 2 – $(7\,6\,4\,8)_{12}$

Sol :

$$1 - 1\,1\,1\,0\,0\,1\,1 = 1 \times 2^0 + 1 \times 2^1 + 0 \times 2^2 + 0 \times 2^3 + 1 \times 2^4 + 1 \times 2^5 + 1 \times 2^6$$
$$= 1 + 2 + 0 + 0 + 16 + 32 + 64$$
$$= (115)_{10}$$
$$(115)_{10} = (0001 \ 0001 \ 0101)_{BCD}$$

Ex What are the Ex-3 of the following numbers?

1 – $(3\,6\,5)_{10}$ 2 – $(1\,1\,0\,1\,0\,1\,0\,1\,0)_2$

Sol

```
   3   6   5
  +3  +3  +3
------------------
   6   9   8
```

Gray Code

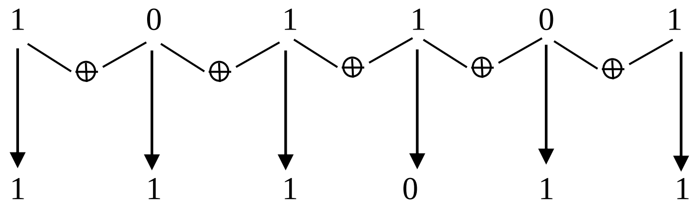
To convert the number from binary to Gray code the relations must be known.

$1 \oplus 1 = 0$
$1 \oplus 0 = 1$
$0 \oplus 1 = 1$
$0 \oplus 0 = 0$

Ex Convert the binary number (1 0 1 1 0 1) into Gray Code?

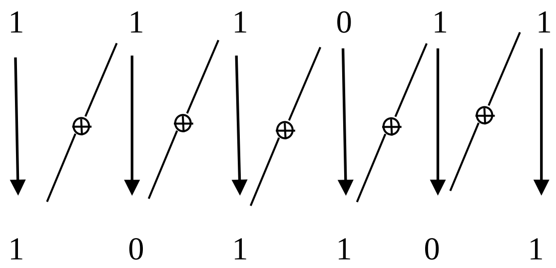
Sol

لتحويل الرقم من صيغة ال Binary الى صيغة Gray يتم انزال ال bit الأول من جهة اليسار ومن ثم تطبق العلاقات السابقة



$(1\ 0\ 1\ 1\ 0\ 1)_2 = (1\ 1\ 1\ 0\ 1\ 1)_G$

لأرجاع الرقم من صيغة Gray الى ال Binary يتم انزال اول bit من جهة اليسار وتطبيق العلاقات بين الناتج والرقم الذي يليه



ASCII Character Code

The standard binary code for the alphanumeric characters is called ASCII (American Standard Code for Information Interchange). It uses 7 bits to code 128 characters, as shown in the table. The seven bits of the code are designed by A_0 through A_6, with A_6 being the most significant bit. For example, the letter A is represented n ASCII as (1000001). The ASCII code contains 94 characters that can print and 34 nonprinting used in control functions. The printing characters consist of 26 uppercase letters, the 26 lowercase letters, 10 numerals, and 32 special printable character such as %, @ and $.

Ex What are the character corresponding to the ASCII code?

(10010101001111100100010011100100000100010010001011010110)$_{ASCII}$

Sol

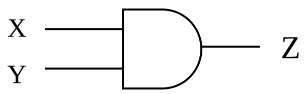
(1001010 1001111 1001000 1001110 0100000 1000100 1000101 1010110)$_{ASCII}$

= JOHN DEV

<u>Binary Logic and Logic Gates</u>

Binary logic is used to describe, in mathematical way, the manipulation and processing of binary information. It is suited for the analysis and design of digital system. The logic circuits (gates) establish the logical manipulation. The logic gates are:-

1 - AND Gate



Logic Symbol

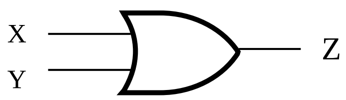
$$Z = X \cdot Y$$

Logic Equation

Input		Output
X	**Y**	**Z**
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table

0   – OR Gate



$$Z = X + Y$$

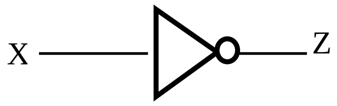
Logic Symbol

Logic Equation

Input		Output
X	**Y**	**Z**
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table

3– Not Gate



Logic Symbol

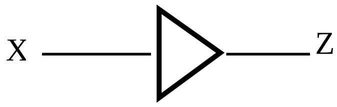
$$Z = \overline{X}$$

Logic Equation

input x	Output z
0	1
1	0

Truth Table

4– Buffer Gate



Logic Symbol

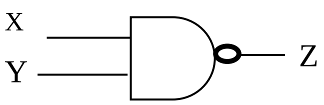
$$Z = X$$

Logic Equation

input x	Output z
0	0
1	1

Truth Table

5 – NAND Gate



Logic Symbol

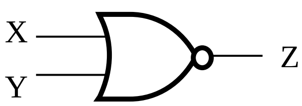
$$Z = \overline{X \cdot Y}$$

Logic Equation

Input X	Input Y	Output Z
0	0	1
0	1	1
1	0	1
1	1	0

Truth Table

6 – NOR Gate



Logic Symbol

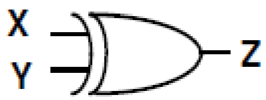
$$Z = \overline{(X + Y)}$$

Logic Equation

Input X	Input Y	Output Z
0	**0**	**1**
0	**1**	**0**
1	**0**	**0**
1	**1**	**0**

Truth Table

7 – Exclusive – OR Gate



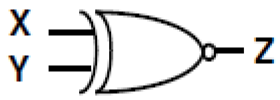
$Z = X \oplus Y$

$= \overline{X}\ Y + X\ \overline{Y}$

Input		Output
X	**Y**	**Z**
0	**0**	**0**
0	**1**	**1**
1	**0**	**1**
1	**1**	**0**

Logic Symbol Logic Equation Truth Table

8 - Exclusive – NOR Gate



$Z = X \bigcirc Y$

$Z = \overline{X \oplus Y}$

$= \overline{X}\ \overline{Y} + X\ Y$

Logic Symbol Logic Equation

Input		Output
X	**Y**	**Z**
0	**0**	**1**
0	**1**	**0**
1	**0**	**0**
1	**1**	**1**

Truth Table

Boolean Algebra

A useful mathematical system for specifying and transforming logic functions, Boolean algebra has six theorems and four postulates

The. 1 a - $X + X = X$ b - $X . X = X$
The. 2 a - $X + 1 = 1$ b - $X . 0 = 0$

The. 3 $(\overline{X})' = X$ involution

The. 4 a – $X + (Y + Z) = (X + Y) + Z$ associative
b - $X(YZ) = (XY)Z$

The. 5 a – $(X + Y)' = \overline{X} . \overline{Y}$
Demorgan theorm
b - $(XY)' = \overline{X} + \overline{Y}$

The. 6 a - $X + XY = X$
Absorption
b - $X(X + Y) = X$

Post. 1 a – $X + 0 = X$ b – $X . 1 = X$

Post. 2 a - $X + \overline{X} = 1$ b - $X \cdot \overline{X} = 0$

Post. 3 a - $X + Y = Y + X$ b – $X Y = Y X$ commutative

Post.4 a $X (Y + Z) = X Y + X Z$

Distributive

b - $X + Y Z = (X + Y) (X + Z)$

Complementing Functions:

Use DeMorgan's Theorem to complement a function:

1.- Interchange AND and OR operators

2.- Complement each constant value and literal

Ex Fined the complement of the functions:-

1 – $F1 = \overline{X} Y \overline{Z} + \overline{X}\ \overline{Y}\ Z$ 2 – $F2 = X (\overline{Y}\ \overline{Z} + Y Z)$

Boolean Function Evaluation

A binary variable can take the value of 0 or 1. a Boolean function is an expression formed with binary variable, the two binary operators OR and AND, the NOT operator, parentheses and equal sign. For a given value of the variables, the function can be either 0 or 1, for example the following Boolean functions:-

$F1 = X \overline{Y} Z$

$F2 = \overline{X} + Y Z$

Any Boolean function can be represented in a truth table. The number of rows in the table is n^2 , where n is the number of binary variables in the function. The binary numbers is then counting from 0 to n^2 -1. The truth table for the above functions:

Inputs			**Outputs**	
X	**Y**	**Z**	**F1**	**F2**
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	
1	1	0	0	
1	1	1	0	

The Boolean function may be transformed to logic diagram composed of AND, OR and NOT gates.

Canonical and Standard Forms

Minterms and Maxterms:

Any Boolean function can be expressed in a canonical form, canonical form include:-

1- Sum of Minterms (SOM)
2- Product of Maxterms (POM)

In Minterms each variable being primed if the corresponding bit of the binary number is 0 and unprimed if a 1.In Maxters each variable being unprimed if the corresponding bit of the binary number is 0 and primed if a 1.

X Y Z	Minterms	Designation	Maxterms	Designation
0 0 0	$\bar{X}\ \bar{Y}\ \bar{Z}$	M0	$X + Y + Z$	M0
0 0 1	$\bar{X}\ \bar{Y}\ Z$	M1	$X + Y + \bar{Z}$	M1
0 1 0	$\bar{X}\ Y\ \bar{Z}$	M2	$X + \bar{Y} + Z$	M2
0 1 1	$\bar{X}\ Y\ Z$	M3	$X + \bar{Y} + \bar{Z}$	M3
1 0 0	$X\ \bar{Y}\ \bar{Z}$	M4	$\bar{X} + Y + Z$	M4
1 0 1	$X\ \bar{Y}\ Z$	M5	$\bar{X} + Y + \bar{Z}$	M5
1 1 0	$X\ Y\ \bar{Z}$	M6	$\bar{X} + \bar{Y} + Z$	M6
1 1 1	$X\ Y\ Z$	M7	$\bar{X} + \bar{Y} + \bar{Z}$	M7

The Sum of Minterms of the function is expressed by thr ORing to the minterms when the output is 1 (F = 1).

The Product of Maxterms of the function is expressed by ANDing to the maxterms when the output is 0 (F = 0).

Ex Find the 1 – Sum of Minterms   2 – Product of Maxterms   of F :-

X Y Z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	0
1 1 0	0
1 1 1	1

Sol:

1 – Sum of Minterms $(\overline{X}\ \overline{Y}\ Z) + (X\ \overline{Y}\ \overline{Z}) + (X\ Y\ Z)$

$= M1 + M4 + M7$

$= \Sigma\,(1, 4, 7)$

2- Product of Maxterms

$$(X + Y + Z) . (X + \overline{Y} + Z) . (X + \overline{Y} + \overline{Z}) . (\overline{X} + Y + \overline{Z}) . (\overline{X} + \overline{Y} + Z)$$

$$= M0 . M2 . M3 . M5 . M6 = \Pi\ (0, 2, 3, 5, 6)$$

Ex Express each of the following functions in Canonical Forms

X Y Z	F1	F2
0 0 0	0	0
0 0 1	1	1
0 1 0	1	0
0 1 1	1	0
1 0 0	0	1
1 0 1	0	1
1 1 0	0	0
1 1 1	1	1

Ex Express the Boolean function $F = A + \overline{B}C$ in Sum of Minterms:-

Sol

The function has three variables A , B , C . the first term missing two variables

$A = A \cdot 1$ by $X \cdot 1 = X$

$= A(B + \overline{B})$ by $X + \overline{X} = 1$

$= AB + A\overline{B}$

$= AB(C + \overline{C}) + A\overline{B}(C + \overline{C}) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\,\overline{C}$

The second term missing one variable

$\overline{B}C = \overline{B}C(A + \overline{A}) = A\overline{B}C + \overline{A}\,\overline{B}C$

$F(A, B, C) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$
$= 111 + 110 + 101 + 100 + 001$

$= m7 + m6 + m5 + m4 + m1 = \Sigma(1, 4, 5, 6, 7)$

Ex Express the Boolean function $F = XY + \overline{X}Z$ in product Maxterms

Sol

First the function must converted into OR terms using distributed low

$F = XY + \overline{X}Z = (XY + \overline{X})(XY + Z) = (X + \overline{X})(Y + \overline{X}) + (X + Z)(Y + Z)$

$= (\overline{X} + Y)(X + Z)(Y + Z)$

Each term is missing one variable

$\overline{X} + Y = \overline{X} + Y + 0$ by $X + 0 = X$

$= \overline{X} + Y + Z\overline{Z}$ by $X \cdot \overline{X} = 0$

$= (\overline{X} + Y + Z)(\overline{X} + Y + \overline{Z})$ by $X + YZ = (X + Y)(X + Z)$

$X + Z = X + Z + 0 = X + Z + Y\overline{Y}$

$= (X + Y + Z)(X + \overline{Y} + Z)$

$Y + Z = Y + Z + 0 = Y + Z + X\overline{X}$

$= (Y + Z + \overline{X})(Y + Z + X)$

$= (\overline{X} + Y + Z)(X + Y + Z)$

$$F = (\overline{X} + Y + Z)(\overline{X} + Y + \overline{Z})(X + Y + Z)(X + \overline{Y} + Z)$$

$$= M4 \,.\, M5 \,.\, M0 \,.\, M2 \quad = \Pi\,(0, 2, 4 .5)$$

Ex Express the Boolean function F in 1- Sum of Minterms 2 – Product of Maxterms

$$F = \overline{A}\ (B + \overline{C})$$

Sol

1 –Sum of Minterms

$$F(A, B, C) = \overline{A}(B + \overline{C}) = \overline{A}B + \overline{A}\,\overline{C}$$

The first term missing one variable

$$\overline{A}B = \overline{A}B.1 = \overline{A}B(C + \overline{C}) = \overline{A}BC + \overline{A}B\overline{C}$$

The second term missing one variable

$$\overline{A}\,\overline{C} = \overline{A}\,\overline{C}.1 = \overline{A}\,\overline{C}(B + \overline{B}) = \overline{A}B\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$$

$$F(A, B, C) = \overline{A}BC + \overline{A}B\overline{C} + \overline{A}\,\overline{B}\,\overline{C} = 011 + 010 + 000 = M3 + M2 + M0$$

$$\sum\ (0, 2, 3)$$

2 – Product of Maxterms

$$F(A, B, C) = \overline{A}\ (B + \overline{C})$$

The first term missing two variables

$$\overline{A} = \overline{A} + 0 = \overline{A} + B\overline{B} = (\overline{A} + B)(\overline{A} + \overline{B})$$

$$= (\overline{A} + B + 0)(\overline{A} + \overline{B} + 0)$$

$$= (\overline{A} + B + C\overline{C})(\overline{A} + \overline{B} + C\overline{C})$$

$$= (\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})$$

The second term missing one variable

$$B + \overline{C} = B + \overline{C} + 0 = B + \overline{C} + A\overline{A} = (B + \overline{C} + A)(B + \overline{C} + \overline{A})$$

$$= (A + B + \overline{C})(\overline{A} + B + \overline{C})$$

$$F(A, B, C) = (\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)(\overline{A} + \overline{B} + \overline{C})(A + B + \overline{C})$$

$$= M4 \,.\, M5 \,.\, M6 \,.\, M7 \,.\, M1 \quad = \Pi\ (1, 4, 5, 6, 7)$$

Ex express the function F in Sum of Minterms and Product of Maxterms and simplify it

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Sol

1– Sum of Minterms

$F(X, Y, Z) = \overline{X}\,Y\,\overline{Z} + \overline{X}\,Y\,Z + X\,\overline{Y}\,Z + X\,Y\,\overline{Z} + X\,Y\,Z$

$= \overline{X}\,Y\,(\overline{Z} + Z) + X\,\overline{Y}\,Z + X\,Y\,(\overline{Z} + Z)$

$= \overline{X}\,Y + X\,\overline{Y}\,Z + XY = Y\,(\overline{X} + X) + X\,\overline{Y}\,Z = Y + X\,\overline{Y}\,Z$

$= X\,\overline{Y}\,Z + Y = (X\,Z + Y)\,(\overline{Y} + Y) = X\,Z + Y$

Standard Forms

Another way to express Boolean function is in standard form. In this form, the terms form the function may contain one, two, or any number of literals, there are two types of standard form:

1– Sum of Product: this expression contain AND terms, called product term, of one or more literals each, and the sum (OR ing) between these terms

Ex $F = \overline{Y} + X\,Y + \overline{X}\,Y\,\overline{Z}$

2– Product of Sum: this expression contain OR terms, called sum term, of one or more literals each, and the product (AND ing) between these terms

Ex $F = X\,.\,(\overline{Y} + Z)\,.\,(\overline{X} + Y + \overline{Z} + W)$

Ex $F = (A\,B + C\,D)\,(\overline{A}\,\overline{B} + \overline{C}\,\overline{D})$

It a non standard form and it can converted to standard form by using distributed law:

$F = A\,B\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,C\,D$