Number systems

In a computer there are many types of number systems, in each type there are the coefficients of the system and the base of the system.

1-Decimal system

This system has ten coefficients (0,1,2,3,4,5,6,7,8,9)

and the base of this system is equal to the number of coefficients, so the base of decimal is(10)

2- Binary system

This system has only two coefficients (0, 1) and the base is (2)

3- Octal system

This system has eight coefficients (0, 1, 2, 3, 4, 5, 6, 7) and the base is(8)

4- Hexadecimal system

The coefficients of this system are (0,1,2,3,4,5,6,7,8,9,A,B,C,E,F) and the base is (16)

Therefore if the **Base** or **Radix** of the system is R then the digits the base is (0,1,2,3,…….R-1) and for the base greater then 10, then symbols are used to represent the digits. In this case, letters, are used to represent the digits greater than (9)

Each number can represent using the rule

$$\ldots\ldots + R^2 \times a_2 + R^1 \times a_1 + R^0 \times a_0 \;.\; b_0 \times R^{-1} + b_1 \times R^{-2} + \ldots\ldots$$

where R is the base of the number system and (a,b) is the digits of the number

Ex

$$(7\ 3\ 9\ 2)_{10} = 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

$$(68\ .\ 53)_{10} = 6 \times 10^1 + 8 \times 10^0\ .\ 5 \times 10^{-1} + 3 \times 10^{-2}$$

ملاحظات

- في اي نظام عددي اساس النظام R والارقام التي تمثل في هذا النظام

  (0, 1, 2, 3, …….R-1)

- اقصى رقم يعطى في النظام هو R-1 ولايجوز تجاوز هذا الرقم
- الأنظمة التي تتجاوز ارقامها ٩ نمثل بلأحرف

  A= 10, B = 11, C=12, D=13, E=14, F=15

وهذا في الأنظمة الحادية عشر و الثانية عشر ..... السادس عشر

Number Base Conversion

A – Conversion from decimal system to another system (Binary , Octal , Hexadecimal ….)

The decimal number is converted to another system by dividing the decimal number by the base of another system and the process is continued until the integer quotient becomes 0. The remainders produced from this process represent the number in another system.

Ex convert decimal 41 to binary?

Sol

		Rem.
41	20	1
20	10	0
10	5	0
5	2	1
2	1	0
1	0	1

$(41)_{10} = (101001)_2$

تؤخذ الارقام بالنظام الجديد من الاسفل الى الاعلى وتكتب من البسار الى البمين

Ex Convert (25) 10 to Binary ?

Sol

		Rem.
25	12	1
12	6	0
6	3	0
3	1	1
1	0	1

$$(25)_{10} = (11001)_2$$

Ex Convert $(53)_{10} \longrightarrow (X)_2$?

Sol

		Rem.
53	26	1
26	13	0
13	6	1
6	3	0
3	1	1
1	0	1

$$(53)_{10} = (110101)_2$$

If the number has a fraction part, then the number is separated into an integer part and fraction part and the conversion of each part done separately.

The fraction part is multiplied by the base of the new system to give an integer and fraction, the new fraction multiplied again to give a new integer and new fraction. The process is continued until the fraction becomes 0. The coefficients of a new number are obtained from the integers.

Ex Convert $(0.6875)_{10}$ to $(X)_2$?

Sol:

0 . 6875 x 2 = 1	. 3750
0 . 3750 x 2 = 0	. 7500
0 . 7500 x 2 = 1	. 5000
0 . 5000 x 2 = 1	.0000

$$(0.6875)_{10} = (0.1011)_2$$

نؤخذ الأرقام (الجزء الصحيح) من الأعلى الى الأسفل وتكتب من اليسار الى اليمين.

Ex Convert $(0.625)_{10}$ to Binary?

Sol:

$0.625 \times 2 = 1$. 250

$0.250 \times 2 = 0$ .500

$0.500 \times 2 = 1$ .000

$(0.625)_{10} = (0.101)_2$

Ex Convert $(17.875)_{10}$ to Binary ?

Sol: 1 – The integer part 2 – the fraction part

		Rem.
17	8	1
8	4	0
4	2	0
2	1	0
1	0	1

$0.875 \times 2 = 1$. 750

$0.750 \times 2 = 1$. 500

$0.500 \times 2 = 1$ .000

$(17.875)_{10} = (10001.111)_2$

Ex Convert $(37)_{10}$ to Octal number?

Sol:

	Rem.
$37 \div 8 = 4$	5
$4 \div 8 = 0$	4

$(37)_{10} = (45)_8$

Ex Convert (95) $_{10}$ to Hexadecimal number?

Sol:

95 ÷ 16 = 5 F ↑

5 ÷ 16 = 0 5

(95) $_{10}$ = (5 F) $_{16}$

Ex Convert (548. 3125) $_{10}$ to Octal number?

Sol:

1 – the integer part

	Rem.
548 ÷ 8 = 68	4 ↑
68 ÷ 8 = 8	4
8 ÷ 8 = 1	0
1 ÷ 8 = 0	1

2 – the fraction part

0 . 3125 x 8 = 2 ↓	. 5000
0 . 5000 x 8 = 4	.0000

(548 , 3125) $_{10}$ = (1044 . 24) $_{8}$

Ex Convert (5213 . 879) $_{10}$ to hexadecimal number?

Sol:

1- The integer part

	Rem.
5213 ÷ 16 = 325	D ↑
325 ÷ 16 = 20	5
20 ÷ 16 = 1	4
1 ÷ 16 = 0	1

2 – The fraction part

. 879 x 16 = 14	. 064	E ↓
, 064 x 16 = 1	. 024	1
. 024 x 16 = 0	. 384	0
. 384 x 16 = 6	.144	6

(5213 . 879) $_{10}$ = (14 5 D . E 1 0 6) $_{H}$

B – Conversion from (Binary , Octal , Hexadecimal …) to Decimal system

In this conversion the following rule is used

$\ldots\ldots + R^2 \times a_2 + R^1 \times a_1 + R^0 \times a_0 \; . \; b_0 \times R^{-1} + b_1 \times R^{-2} + b_2 \times R^{-3} \ldots\ldots$

Ex Convert $(110111)_2$ to Decimal number?

Sol:

$110111 = 1 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 1 \times 2^4 + 1 \times 2^5$

$= 1 + 2 + 4 + 0 + 16 + 32$

$= (55)_{10}$

Ex Convert $(11010 . 11)_2$ to Decimal ?

Sol:

1 – The integer part

$11010 = 0 \times 2^0 + 1 \times 2^1 + 0 \times 2^2 + 1 \times 2^3 + 1 \times 2^4$

$= 0 + 2 + 0 + 8 + 16 = 26$

2 – the fraction part

$. 11 = 1 \times 2^{-1} + 1 \times 2^{-2} = 0 . 5 + 0 . 25 = 0. 75$

$(11010 . 11)_2 = (26 . 75)_{10}$

Ex Convert $(3624)_8$ to Decimal number ?

Sol:

$3624 = 4 \times 8^0 + 2 \times 8^1 + 6 \times 8^2 + 3 \times 8^3$

$= 4 + 16 + 6 \times 64 + 3 \times 512$

$= 4 + 16 + 384 + 536 = (1940)_{10}$

Ex Convert $(A 2 F)_H \longrightarrow (X)_{10}$?

Sol :

$A\ 2\ F = 15 \times 16^0 + 2 \times 16^1 + 10 \times 16^2$

$= 15 + 32 + 10 \times 256$

$= 15 + 32 + 2560 = 2607$

$(A\ 2\ F)_H = (2\ 6\ 0\ 7)_{10}$

Ex Convert $(2\ 2\ 2\ 1)_3$ to $(X)_{10}$?

Sol :

$2\ 2\ 2\ 1 = 1 \times 3^0 + 2 \times 3^1 + 2 \times 3^2 + 2 \times 3^3$

$= 1 + 6 + 2 \times 9 + 2 \times 27$

$= 1 + 6 + 18 + 54 = 79$

$(2\ 2\ 2\ 1)_3 = (79)_{10}$

C – Conversion between the number system:

1 – Conversion between binary and Octal number

Octal no.	Binary D_2	D_1	D_0
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

Conversion from binary to octal number is done by partitioning the binary number into groups of three digits; the corresponding octal digit is then assigned to each group.

Ex Convert $(1\ 1\ 1\ 0\ 1)_2 \longrightarrow (X)_8$?

Sol:

$(0\ 1\ 1\ \ 1\ 0\ 1)_2 = (35)_8$

Ex Convert $(1\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ .\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1)_2 \longrightarrow (X)_8$?

Sol:

$(101\ 100\ 011\ 010\ 111\ .\ 111\ 100\ 000\ 111)_2 = (54327.7407)_8$

Conversion the Octal number to Binary number is done by convert each octal digit to its three digit binary equivalent.

Ex Convert the following numbers?

1- $(123.4)_8 \longrightarrow (X)_2$

2 - $(572.134)_8 \longrightarrow (X)_2$

Sol : $(123.4)_8 = (001\ 010\ 011\ .\ 100)_2$

2 – Conversion between binary and Hexadecimal number

Hexadecimal No.	Binary D_3	D_2	D_1	D_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
A	1	0	1	0
B	1	0	1	1
C	1	1	0	0
D	1	1	0	1
E	1	1	1	0
F	1	1	1	1

Conversion from binary to Hexadecimal number is done by partitioning the binary number into groups of four digits; the corresponding Hexadecimal digit is then assigned to each group.

Ex Convert $(1110111011111)_2 \longrightarrow (X)_H$?

Sol **:**

$(00\ 11\ 1011\ 1101\ 1111\ )_2 \longrightarrow (2BDF)_H$

Ex Convert $(110011011.110101)_2 \longrightarrow (X)_H$?

Sol:

$(0001\ 1001\ 1011\ .\ 1101\ 0100)_2 \longrightarrow (19B.D4)_H$

Conversion the Hexadecimal number to Binary number is done by convert each Hexadecimal digit to its four digit binary equivalent

.

Ex Convert the following numbers?

1- $(95.6)_{16} \longrightarrow (X)_2$

2 - $(A05.963)_{16} \longrightarrow (X)_2$

- $(A05.963)_{16} = (1010\ 0000\ 0101.1001\ 0110\ 0011)_2$

3 – Conversion from Hexadecimal number to Octal number

- ***Convert the Hexadecimal number to Binary number***
- ***Convert the Binary number to Octal number***

Ex Convert $(A6.D1)_{16} \longrightarrow (X)_8$?

Sol:

- $(A6.D1)_{16} = (1010\ 0110\ .1101\ 0001)_2$

- $(010\ 100\ 110\ .110\ 100\ 010)_2 = (246\ .\ 642)_8$

4 – Conversion from Octal number to Hexadecimal number

- ***Convert the Octal number to Binary number***
- ***Convert the Binary number to Hexadecimal number***

Ex Convert $(716.25)_8 \longrightarrow (X)_{16}$?

Sol:

$(716.25)_8 = (111\ 001\ 110\ .\ 010\ 101)_2$

$(0001\ 1100\ 1110\ .\ 0101\ 0100)_2 = (1CE.54)_{16}$

ملاحظة: هناك خالات تحويل بين انظمة مختلفة مثال على ذلك

$(X)_5 \longrightarrow (X)_8$ or $(X)_2 \longrightarrow (X)_7$

في هذه الحالة يتم التحويل الى النظام العشري ثم يحول الرقم الناتج الى النظام المطلوب

Ex Convert $(1\,1\,0\,1\,1\,0)_2 \longrightarrow (X)_7$?

Sol:

1 - $(1\,1\,0\,1\,1\,0) = 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 1 \times 2^4 + 1 \times 2^5$

$= 0 + 2 + 4 + 0 + 16 + 32 = (54)_{10}$

2 -

	Rem.
$54 \div 7 = 7$	5
$7 \div 7 = 1$	0
$1 \div 7 = 0$	1

$(1\,1\,0\,1\,1\,0)_2 = (54)_{10} = (1\,0\,5)_7$

Arithmetic Oprations العمليات الحسابية

Binary arithmetic operations

1 – Addition

0 + 0 = 0
0 + 1 = 1
1 + 0 = 1
1 + 1 = 0 and carry 1
1 + 1 + 1 = 1 and carry 1

2 – Subtraction

0 - 0 = 0
1 - 0 = 1
1 - 1 = 0
0 - 1 = 1 and barrow 1 from next column

3 – Multiplication

0 x 0 = 0
0 x 1 = 0
1 x 0 = 0
1 x 1 = 1

4 – Division

0 ÷ 0 = 0

0 ÷ 1 = 0
1 ÷ 1 = 1
1 ÷ 0 = overflow

Ex Perform the following operations
1-(1 1 1 1)$_2$ + (1 0 1 0 0)$_2$ 2-(1 1 . 0 1)$_2$ + (1 0 1 . 1 1)$_2$
3-(1 0 1 1 0 1)$_2$ + (1 0 0 1 1 1 1)$_2$

Sol:

1-
```
    1 1 1 1
+   1 0 1 0 0
-------------
1 0 0 0 1 1
```

2-
```
    1 1 . 0 1
+   1 0 1 . 1 1
--------------
1 0 0 1 . 0 0
```

Ex Subtract (1 0 1)$_2$ from (1 0 0 1)$_2$?

Sol:

1 0 0 1 - 1 0 1 = (0 1 0 0)$_2$

Ex Subtract (1 1 0 . 0 1)$_2$ -(1 0 0 . 1)$_2$?

Sol:

1 1 0 . 0 1 - 1 0 0 . 1 = (0 0 1 . 1 1)$_2$

H.W.

1-(1 0 1 1 0 1)$_2$ -(1 0 0 1 1 1)$_2$
2- (1 1 1 1)$_2$ -(1 1 0 0)$_2$
3- (0 . 1 1 0)$_2$ -(0 . 0 1 1 0)$_2$
4-(1 1 1 0 1 . 0 1)$_2$ -(1 0 0 1 1 . 1)$_2$

Ex Perform the following operations:-
1- (1 0 1 1)$_2$ x (1 0 1)$_2$ 2-(1 . 0 1)$_2$ -(1 0 . 1)$_2$

Sol:

```
      1 0 1 1
x     1 0 1
-------------
```

```
      1 . 0 1
x   1 0 . 1
---------------
```

```
    1 0 1 1
+   0 0 0 0
  1 0 1 1
-----------
1 1 0 1 1 1
```

```
      1 0 1
+     0 0 0
    1 0 1
-----------
  1 1 . 0 0 1
```

Ex : Perform the following operations:

1 - $(1111)_2 \div (101)_2$

2 - $(11001)_2 \div (101)_2$

3 - $(10110)_2 \div (10)_2$

4 - $(11011)_2 \div (100)_2$

5 - $(11101)_2 \div (1100)_2$

6 - $(100100001)_2 \div (1011)_2$

7 - $(1010.01)_2 \div (1.1)_2$

Sol:

1–

```
          1 1
      ---------
101 | 1 1 1 1
      1 0 1
    -----------
      0 1 0 1
        1 0 1
    -----------
        0 0 0
```

2 -

```
          1 0 1
      -----------
101 | 1 1 0 0 1
      1 0 1
    -------------
      0 0 1 0 1
        1 0 1
    -------------
        0 0 0
```