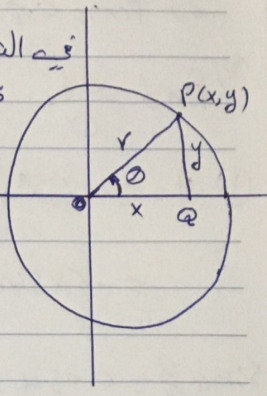
Trigonometric Functions (الدوال المثلثية)

في الشكل التالي نلاحظ الدائرة التي نصف قطرها r ومركزها o (0,0) وكذلك الزاوية المرتسمة θ تقابل الضلع القائم y و المجاور x من المثلث oPQ نعرف النسب المثلثية لـ θ بالعلاقات التالية.



① $\sin\theta = \frac{y}{r} = \frac{\text{المقابل}}{\text{الوتر}}$

$\cos\theta = \frac{x}{r} = \frac{\text{المجاور}}{\text{الوتر}}$

$\tan\theta = \frac{y}{x}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$

Derivative of Trigonometric functions.

if ① $f(x) = \sin x$ then $f'(x) = \cos x$

② $f(x) = \cos x$ then $f'(x) = -\sin x$

③ $f(x) = \tan x$ then $f'(x) = \sec^2 x$

④ $f(x) = \sec x$ then $f'(x) = \tan x \cdot \sec x$

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5) $f(x) = \csc x$, then $f'(x) = -\csc x \cot x$

6) $f(x) = \cot x$, then $f'(x) = -\csc^2 x$

② Find $\frac{dy}{dx}$

① $y = \cos(3x^2+1) \Rightarrow \frac{dy}{dx} = -\sin(3x^2+1) \times 6x$

$= -6x \sin(3x^2+1)$

since $y = \cos u x$, $y' = -\sin u x \cdot \frac{du}{dx}$

② $y = x^3 \sin^2(5x) \Rightarrow \frac{dy}{dx} = x^3 \cdot \frac{d}{dx}[\sin^2 5x] + \sin^2 5x \cdot \frac{d}{dx} x^3$

$= 2x^3 \sin 5x \cdot \frac{d}{dx}[\sin 5x] + \sin^2 5x \, (3x^2) \cdot$

$= 2x^3 \sin 5x \cdot (\cos 5x) \cdot 5 + 3x^2 \sin^2 5x \cdot$

$= 10 x^3 \sin 5x \cos 5x + 3x^2 \sin^2 5x \cdot$

③ $y = 2\tan(\frac{x}{2}) - x \Rightarrow \frac{dy}{dx} = 2\sec^2(\frac{x}{2}) \cdot \frac{1}{2} - 1$

$= \sec^2(\frac{x}{2}) - 1$

④ $y = \sec^4(3x) \Rightarrow y' = 4\sec^3(3x) \cdot \sec(3x)\tan(3x) \cdot 3$

$= 12 \sec^4(3x) \cdot \tan(3x)$

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5) $y = 3\sqrt{\cot(3x)} \Rightarrow y = 3(\cot 3x)^{\frac{1}{2}}$

$$y' = \frac{dy}{dx} = 3 \cdot \frac{1}{2}(\cot 3x)^{\frac{-1}{2}} \cdot (-\csc^2 3x \cdot 3)$$

$$= \frac{-9}{2} \frac{\csc^2 3x}{\sqrt{\cot 3x}}$$

$\frac{d}{dx}(\cos 2x)$

$y = \cos^4 2x \Rightarrow y' = 4\cos^3 2x \cdot (-\sin 2x \cdot 2)$

$$= -8\cos^3 2x \sin 2x$$

6) $y = \sqrt{x^3 + \csc 2x}$

$$y' = \frac{dy}{dx} = \frac{1}{2}(x^3 + \csc 2x)^{-\frac{1}{2}} \cdot (3x^2 - 2\csc 2x \cot 2x)$$

$$= \frac{(3x^2 - 2\csc 2x \cot 2x)}{2(x^3 + \csc 2x)^{\frac{1}{2}}}$$

7) $y = \sin(\sqrt{1 + \cos(10x)}) \Rightarrow y' = \cos(\sqrt{1 + \cos(10x)}) \cdot \frac{d}{dx}(\sqrt{1 + \cos 10x})$

~~$= \cos(\sqrt{1 + \cos(10x)}) \cdot \frac{1}{2}(1 + \cos 10x)^{\frac{1}{2}} \cdot (-10\sin 10x)$~~

$$= \cos(\sqrt{1 + \cos 10x}) \cdot \frac{1}{2}(1 + \cos 10x)^{\frac{1}{2}} \cdot (-10\sin 10x)$$

$$= \frac{\cos(\sqrt{1 + \cos 10x})(-10\sin 10x)}{2(1 + \cos 10x)^{\frac{1}{2}}}$$

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Exponetial functions (e^x)

Derivatives of Exponetial and Logarithmic funct.

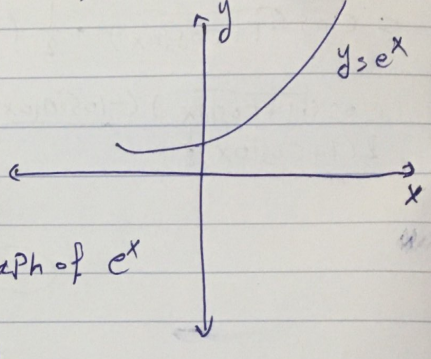
Rules of e^x

① $e^x \cdot e^y = e^{x+y}$ ② $\frac{e^x}{e^y} = e^{x-y}$

③ $e^{-x} = \frac{1}{e^x}$ ④ $\ln e^x = x$, $e^{\ln x} = x$

1) $e^x = R$

$Re^x = \{ y : y > 0 \}$



Graph of e^x

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Derivative of e^x

$$\frac{d}{dx}(e^{f(x)}) = e^{f(x)} * f'(x).$$

* يكون الاشتقاق لدالة e^x هو (نفس الدالة الأسية) في مشتقة الأس.

Ex) ① find $\frac{dy}{dx}$

a) $y = e^{3x} \Rightarrow \frac{dy}{dx} = y' = e^{3x} * 3 = 3e^{3x}$

b) $y = e^{\sin x} \Rightarrow y' = e^{\sin x} * \cos x$

c) $y = e^{\sqrt{1+5x^2}} \Rightarrow y' = e^{\sqrt{1+5x^2}} * \frac{10x}{2\sqrt{1+5x^2}}$ ($(\frac{1}{2}(1+5x^2)^{\frac{1}{2}} \times 10x)$)

d) $y = e^{-x/6}\cos 4x \Rightarrow y' = \frac{d}{dx}(e^{-x/6}\cos 4x)$

$= e^{-x/6}(-4\sin 4x) + (-\frac{1}{6}e^{-x/6})\cos 4x$

$= -4e^{-x/6}\sin 4x - \frac{1}{6}e^{-x/6}\cos 4x$

e) $y = \sec^2(e^{4x}) \Rightarrow \frac{dy}{dx} = 2\sec(e^{4x}) * \sec(e^{4x}) \cdot \tan(e^{4x}) * e^{4x} \cdot 4$

$= 8e^{4x}\sec^2(e^{4x}) \cdot \tan(e^{4x})$.

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① $\ln(xy) = \ln x + \ln y$

② $\ln(x/y) = \ln x - \ln y$

③ $\ln x^3 = 3 \ln x$

④ $\ln e^x = x = e^{\ln x} = x$

Deriv. of $\ln u$

$$\frac{d}{dx}(\ln u) = \frac{1}{u} * \frac{du}{dx}$$

Ex: Find y' for

① $y = \ln x^4 \Rightarrow y = 4\ln x$

$$\frac{dy}{dx} = 4 * \frac{1}{x} * 1 = \frac{4}{x}$$

② $y = \ln \cos x \Rightarrow \frac{dy}{dx} = \frac{1}{\cos x} * -\sin x$

$$= \frac{-\sin x}{\cos x} = -\tan x$$

③ $y = \ln (x^2 + \cos x)$

$$y' = \frac{1}{x^2 + \cos x} * (2x - \sin x) = \frac{(2x - \sin x)}{x^2 + \cos x}$$

④ $y = \tan(\ln x) \Rightarrow y' = \sec^2(\ln x) * \frac{1}{x}$

⑤ $y = \ln (x^2+2)^2 * \cos x \Rightarrow y = 2\ln(x^2+2) * \cos x$

$$y' = 2\ln(x^2+2) * -\sin x + \cos x \left(2 \frac{*2x}{(x^2+2)}\right)$$

$$= -2\sin x \ln(x^2+2) + \frac{4x \cos x}{x^2+2}$$

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