Lecture (21)

Grid Staggering

**21.1 Introduction**

There are two kinds of grid arrangements: unstaggered grids and staggered grids. On a staggered grid the scalar variables (pressure, density, total enthalpy etc.) are stored in the cell centers of the control volumes, whereas the velocity or momentum variables are located at the cell faces. This is different from a [collocated grid](http://www.cfd-online.com/Wiki/Collocated_grid) arrangement (unstaggered grids), where all variables are stored in the same positions.

Using a staggered grid is a simple way to avoid [odd-even decoupling](http://www.cfd-online.com/W/index.php?title=Odd-even_decoupling&action=edit&redlink=1) between the pressure and velocity. Odd-even decoupling is a discretization error that can occur on collocated grids and which leads to checkerboard patterns in the solutions.

The disadvantage of using staggered grids is that different variable are stored at different places and this makes it more difficult to handle different control volumes for different variables.

The use of unstaggered grids greatly reduces the required storage memory and shortens the computational time in three-dimensional calculations. However, they are prone to produce a false pressure field-checkerboard pressure. For this reason, in the 1980s and before, unstaggered grids were rarely used in the primitive variable method for incompressible flow. However, since 1983 the unstaggered grid (or collocated grid) has been used again more and more widely, after the scientists Rhie and Chow (1983) proposed a momentum interpolation method to eliminate the checkerboard pressure.

Figure (3.2) shows a one-dimensional schematic of an approach of staggering. For the unstaggered grid shown in figure (3.2a), calculation of an advection term such as u∂θ/∂x with a centered, three-point method would require differencing across a 2∆x interval. For the staggered grid in figure 3.2b, the derivative can be calculated by differencing across a 1∆x interval. This halves the effective grid increment for such terms, increasing the spatial resolution and decreasing the effects of truncation error on the solution.

$u\_{i+1}, θ\_{i+1}$

$u\_{i-1}, θ\_{i-1}$

$u\_{i}, θ\_{i}$

$∆x$

$u\_{i+1}$

$ θ\_{i+{1}/{2}}$

$u\_{i}$

$θ\_{i-{1}/{2}}$

$u\_{i-1}$

$(a)$

$(b)$

Figure 3.2. Schematic of one-dimensional unstaggered (a) and staggered (b) grids. For the staggered grid, the mass-field variable (θ) is offset by one-half grid increment from the momentum variable (u)

Let's consider the shallow water equations (SWE) in two dimensions:

$$ \frac{∂h}{∂t}=-\left[h(\frac{∂u}{∂x}+\frac{∂v}{∂y})\right]-u\frac{∂h}{∂x}-v\frac{∂h}{∂y} (3.48)$$

$$ \frac{∂u}{∂t}=-\left[g\frac{∂h}{∂x}+fv\right]-u\frac{∂u}{∂x}-v\frac{∂u}{∂y} (3.49)$$

$$ \frac{∂v}{∂t}=-\left[g\frac{∂h}{∂y}+fu\right]-u\frac{∂v}{∂x}-v\frac{∂v}{∂y} (3.50)$$

The terms in square brackets in (3.48-3.50) are the dominant terms for the geostrophic and the inertia-gravity wave dynamics. These terms are computed in different ways depending on the type of grid used. The advective terms are less affected by the choice of alternative (staggered) grids.

**21.2 Types of Staggered Grids**

In two dimensions there are several possibilities for staggered grids (Arakawa and Lamb, 1977), which are shown in Figure (3.3). Grid A (unstaggered) has several advantages and disadvantages. The advantages are its simplicity, and, because all variables are available at all the grid points, it is easy to construct a higher order accuracy scheme. Grid A tends to be favored by proponents of the philosophy “accuracy is more important than conservation”. Its main disadvantage is that all differences occur on distances 2∆, and that neighboring points are not coupled for the pressure and convergence terms. This can give rise in time to a horizontal uncoupling (checkerboard pattern), which needs to be controlled by using a high order diffusion.

Grid C (Arakawa C-Grid) has the advantage that the convergence and pressure terms in square brackets in (3.48-3.50) are computed over a distance of only 1∆, which is equivalent to doubling the resolution of grid A. For this reason geostrophic adjustment (the dispersion of gravity waves generated when the fields are not in geostrophic balance) is computed much more accurately.

*Staggered grid D*

*Staggered grid E*

*Staggered grid C*

*Staggered grid B*

*u v h*

*Unstaggered grid A*

*u v*

*u v h*

*u v*

*u v*

*u v*

*u*

*h*

*v*

*v*

*u*

*v*

*h*

*u*

*u*

*v*

*u v*

*uv*

*h*

*uv*

*uv*

*uv*

Figure 3.3. Staggered grids in two horizontal dimensions

The B and E grids carry both wind components at the same points, and are referred to as semi-staggered. The C and D grids provide two fully staggered layouts in which there are different locations for the two wind components and for height. Note that the E grid can be viewed as the B grid rotated through 45o (Simmons, 1994). The staggered grid has hence the advantage that: (i) the computational time is half as long or the number of grid points is halved, (ii) the truncation error is half as big with $∆x\rightarrow ∆x/2$, (iii) waves with $k∆x>π/2,$ which are shorter than 4 grid cells, are eliminated. They are the ones that have a big error in the phase speed (Döös, 2011). The problems on the staggered grid arise due to the averaging of the velocity components in the Coriolis force terms (Janjic et al. 2010). The momentum components (u and v) are located together on what will be called the momentum or velocity grid. Surface pressure, temperature, and an arbitrary number of tracers are located together on what will be called the mass or temperature grid. The momentum and mass grids are rectangular in shape, with equal spacing in longitude along the x-axis and latitude along the y-axis. The grids are diagonally shifted from each other, such that, the center of a momentum grid box is located at the corner where four mass

grid boxes intersect. Auxiliary grids can be defined for computing additional quantities. The zonal mass flux grid (or U grid) has grid boxes centered on the east and west faces of a mass grid box. The meridional mass flux grid (or V grid) has grid boxes centered on the north and south faces of a mass grid box (Wyman, 2003). It is well known for finite-difference models that the geostrophic adjustment and Rossby wave propagation are better reproduced by the unstaggered grid combined with the vorticity-divergence formulation than by the standard *u*–*v* formulation on the staggered C grid. The unstaggered grid applied to the semi-Lagrangian model also allows use of a single set of trajectories for all variables (while for a staggered grid it is necessary either to use the multiple set of trajectories or to make additional interpolations). On the other hand, the unstaggered grid makes it possible to apply easily compact high- order finite differences. Otherwise, one would need to apply high-order interpolation between half- and integer nodes of the grid (for example, to calculate the Coriolis term on the C grid) (Tolstykh, 2002).