Lecture (6)

The Effect of Changing Time Step of Explicit scheme

6.1 The Effect of Changing Time Step

In this lecture, we will examine the effect of changing time step on the solution of the partial differential equations by the explicit method. Returning to example (5.2) in the previous lecture, and taking into account the following values:

h=1/10 , k = 5/1000 (r=1/2) so the equation:

$$u_{i,j+1} = u_{i,j} + r\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right) \qquad (6.1)$$

will turn to:

$$u_{i,j+1} = \frac{1}{2}\left(u_{i-1,j} + u_{i+1,j}\right) \qquad (6.2)$$

The solution to be found by applying the finite difference equation (6.2) on the boundary and initial conditions is listed in Table (6.1). The percentage errors are listed in Table (6.2)

Table (6.1)

i=0	x = 0	i= 1 0.1	i= 2 0.2	i= 3 0.3	i =4 0.4	i= 5 0.5	i= 6 0.6
t=0.000	**0**	**0.2000**	**0.4000**	**0.6000**	**0.8000**	**1.0000**	**0.8000**
0.005	**0**	**0.2000**	**0.4000**	**0.6000**	**0.8000**	**0.8000**	**0.8000**
0.010	**0**	**0.2000**	**0.4000**	**0.6000**	**0.7000**	**0.8000**	**0.7000**
0.015	**0**	**0.2000**	**0.4000**	**0.5500**	**0.7000**	**0.7000**	**0.7000**
0.020	**0**	**0.2000**	**0.3750**	**0.5500**	**0.6250**	**0.7000**	**0.6250**
⋮							
0.020	**0**	**0.0949**	**0.1717**	**0.2484**	**0.2778**	**0.3071**	**0.2778**

Table (6.2)

	Finite –difference Solution (at x=0.3)	Analytical solution (at x=0.3)	Difference	Percentage error
t=0.005	**0.6000**	**0.5966**	**0.0034**	**0.57**
t=0.01	**0.6000**	**0.5799**	**0.0201**	**3.5**
t=0.02	**0.5500**	**0.5334**	**0.0166**	**3.1**
t=0.1	**0.2484**	**0.2444**	**0.0040**	**1.6**

It is clear that the finite difference solution is not a good approximation for the PDE as in the previous case (example 5.2), but it is acceptable for technical purposes.

Now, if we use h=1/10, k= 1/100 i.e. r = 1 then equation 6.1 will be:

$$u_{i,j+1} = u_{i-1,j} - u_{i,j} + u_{i+1,j} \qquad (6.3)$$

The solution will be as in Table (6.3)

Table (6.3)

	i=0 x = 0	i= 1 0.1	i= 2 0.2	i= 3 0.3	i =4 0.4	i= 5 0.5	i= 6 0.6
t=0.000	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**0.8**
0.01	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**0.6**	**0.8**
0.02	**0**	**0.2**	**0.4**	**0.6**	**0.4**	**1.0**	**0.4**
0.03	**0**	**0.2**	**0.4**	**0.2**	**1.2**	**-0.2**	**1.2**
0.04	**0**	**0.2**	**0.0**	**1.4**	**-1.2**	**2.6**	**-1.2**

For a partial differential equation, this solution is meaningless even though it is true solution for equation (6.3). The three cases (k= 1/1000, k= 5/1000, and k=1/100) show that the acceptable value of (r) for an explicit method is : $0 \le r \le \frac{1}{2}$.

Figure 6.1 shows a comparison between the analytical solution for PDE (continuous curves) and the finite difference solution of PDE (dots) for r value under 0.5 and above 0.5 with the same number of time steps.

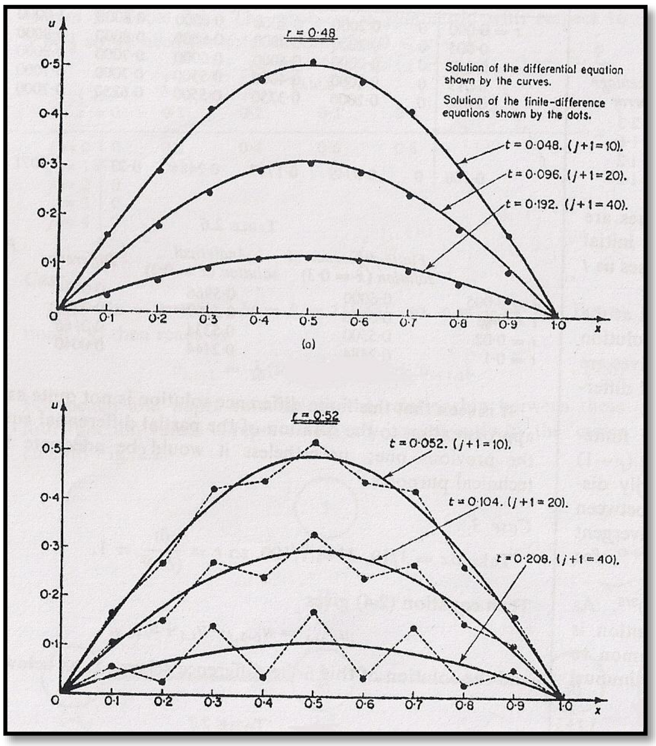

Figure (6.1) Comparison between the analytical and finite difference solutions

6.2 Exercise: Explain the effect of changing the time step on the accuracy of the finite difference solution.