



State Space Search: Water Jug Problem

“You are given two jugs, a 4-litre one and a 3-litre one. Neither has any measuring markers on it. There is a pump that can be used to fill the jugs with water. How can you get exactly 2 litres of water into 4-litre jug.”

Ismael Abdul
AL-Mstansiriyah University
College of science

State Space Search: Water Jug Problem

- State: (x, y)
 $x = 0, 1, 2, 3, \text{ or } 4$ $y = 0, 1, 2, 3$
- Start state: $(0, 0)$.
- Goal state: $(2, n)$ for any n . such that $n \leq 3$
- Attempting to end up in a goal state.

State Space Search: Water Jug Problem

1. (x, y) $\rightarrow (4, y)$ Fill the 4-gallon jug
if $x < 4$
2. (x, y) $\rightarrow (x, 3)$ Fill the 3-gallon jug
if $y < 3$
3. (x, y) $\rightarrow (x - d, y)$ Pour some water out of the 4-gallon jug
if $x > 0$
4. (x, y) $\rightarrow (x, y - d)$ Pour some water out of the 3-gallon jug
if $y > 0$

State Space Search: Water Jug Problem

5. $(x, y) \rightarrow (0, y)$
if $x > 0$

Empty the 4-gallon jug on the ground

6. $(x, y) \rightarrow (x, 0)$
if $y > 0$

Empty the 3-gallon jug on the ground

7. $(x, y) \rightarrow (4, y - (4 - x))$
if $x + y \geq 4, y > 0$

Pour water from the 3-gallon jug into the 4-gallon jug until the 4-gallon jug is full

8. $(x, y) \rightarrow (x - (3 - y), 3)$
if $x + y \geq 3, x > 0$

Pour water from the 4-gallon jug into the 3-gallon jug until the 3-gallon jug is full

State Space Search: Water Jug Problem

9. $(x, y) \rightarrow (x + y, 0)$
if $x + y \leq 4, y > 0$

Pour all the water from the 3-gallon jug into the 4-gallon jug

10. $(x, y) \rightarrow (0, x + y)$
if $x + y \leq 3, x > 0$

Pour all the water from the 4-gallon jug into the 3-gallon jug

11. $(0, 2) \rightarrow (2, 0)$

Pour 2-gallons from the 3-gallon jug into the 4-gallon jug

12. $(2, y) \rightarrow (0, y)$

Empty the 2-gallons in the 4-gallon jug on the ground

One solution to the water jug problem

Gallons in the 4-gallon jug	Gallons in the 3-gallon jug	Rule Applied
0	0	2
0	3	9
3	0	2
3	3	7
4	2	5 or 12
0	2	9 or 11
2	0	

Another water jug problem

Problem statement : we have 2 jugs, a 5-gallon (5-g) and the other 3-gallon (3-g) with no measuring marker on them. There is endless supply of water through tap. our task is to get 4-gallon of water in the 5-g jug

Solution : state space of this problem can be described as the set of ordered pairs of integers (X,Y) such that X represents the number of gallons of water in 5-g jug and Y for 3-g jug.

1. Start state is $(0,0)$
2. goal state is $(4,n)$ for any value of $n \leq 3$

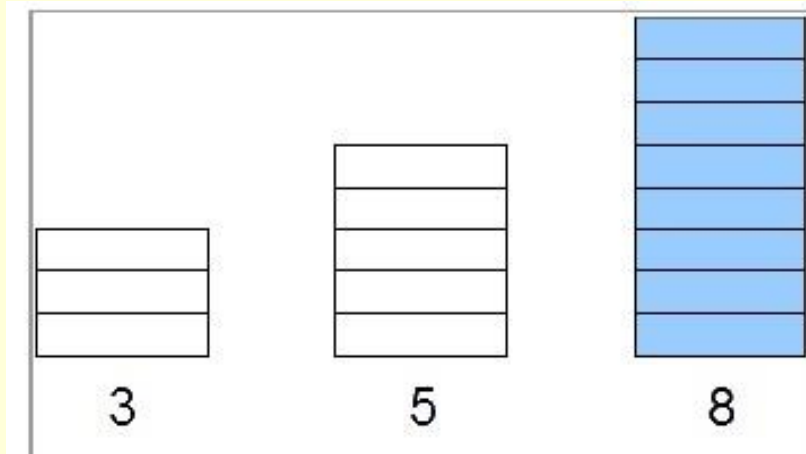
Rule No	Left of rule	Right of rule	Description
1	$(X, Y \mid X < 5)$	$(5, Y)$	Fill 5-g jug
2	$(X, Y \mid X > 0)$	$(0, Y)$	Empty 5-g jug
3	$(X, Y \mid Y < 3)$	$(X, 3)$	Fill 3-g jug
4	$(X, Y \mid Y > 0)$	$(X, 0)$	Empty 3-g jug
5	$(X, Y \mid X + Y \leq 5 \wedge Y > 0)$	$(X + Y, 0)$	Empty 3-g into 5-g jug
6	$(X, Y \mid X + Y \leq 3 \wedge X > 0)$	$(0, X + Y)$	Empty 5-g into 3-g jug
7	$(X, Y \mid X + Y \geq 5 \wedge Y > 0)$	$(5, Y - (5 - X))$ until 5-g jug is full	Pour water from 3-g jug into 5-g jug
8	$(X, Y \mid X + Y \geq 3 \wedge X > 0)$	$(X - (3 - Y), 3)$	Pour water from 5-g jug into 3-g jug until 3-g jug is full

Rule applied	5-g jug	3-g jug	Step No
Start state	0	0	
1	5	0	1
8	2	3	2
4	2	0	3
6	0	2	4
1	5	2	5
8	4	3	6
Goal state	4	—	

Ismael Abdul
AL-Mstansiryiah University
College of science

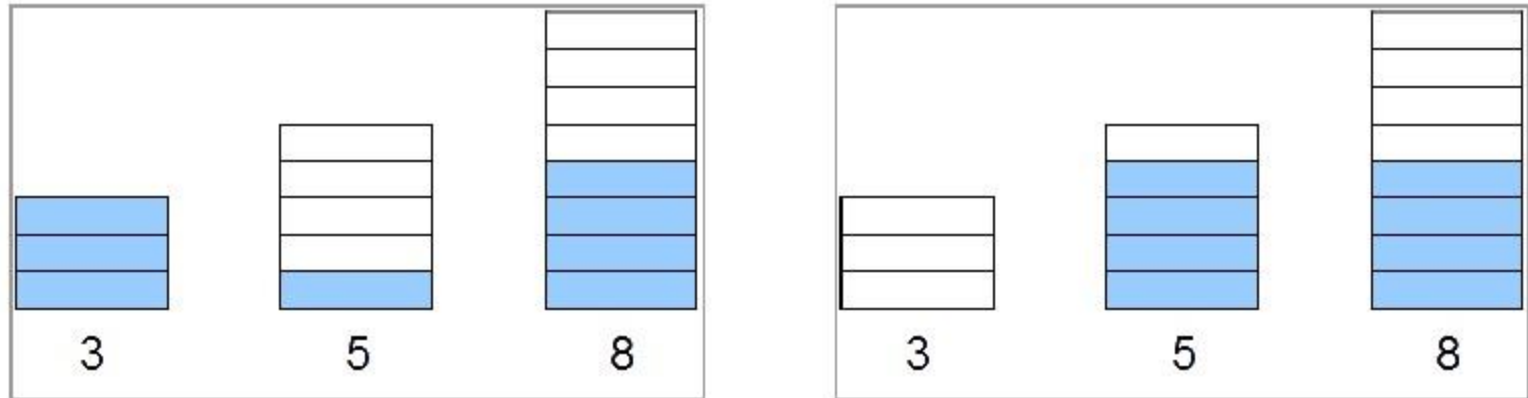
Rule applied	5-g jug	3-g jug	Step No
Start state	0	0	
3	0	3	1
5	3	0	2
3	3	3	3
7	5	1	4
2	0	1	5
5	1	0	6
3	1	3	7
5	4	0	8
Goal state	4	—	

We have 3 jugs of capacities 3, 5, and 8 liters, respectively. There is no scale on the jugs, so it's only their capacities that we certainly know. Initially, the 8-litre jug is full of water, the other two are empty:



This lines just for visualization because there is nom measuring mark on the jugs

We can pour water from one jug to another, and the goal is to have exactly 4 liters of water in any of the jugs. The amount of water in the other two jugs at the end is irrelevant. Here are two of the possible goal states:



Give the following :

- Set of states
- Initial state
- Set of goal states
- Set of operators
- Precondition of the operators
- State space graph

Farmer problem

The problem may be stated as follows. A **farmer** wants to transfer his three belongings, a **wolf**, a **goat** and a **cabbage**, by a boat from the left bank of a river to its right bank. The boat can carry at most two items including the farmer. If unattended, the wolf may eat up the goat and the goat may eat up the cabbage. **How should the farmer plan to transfer the items?**

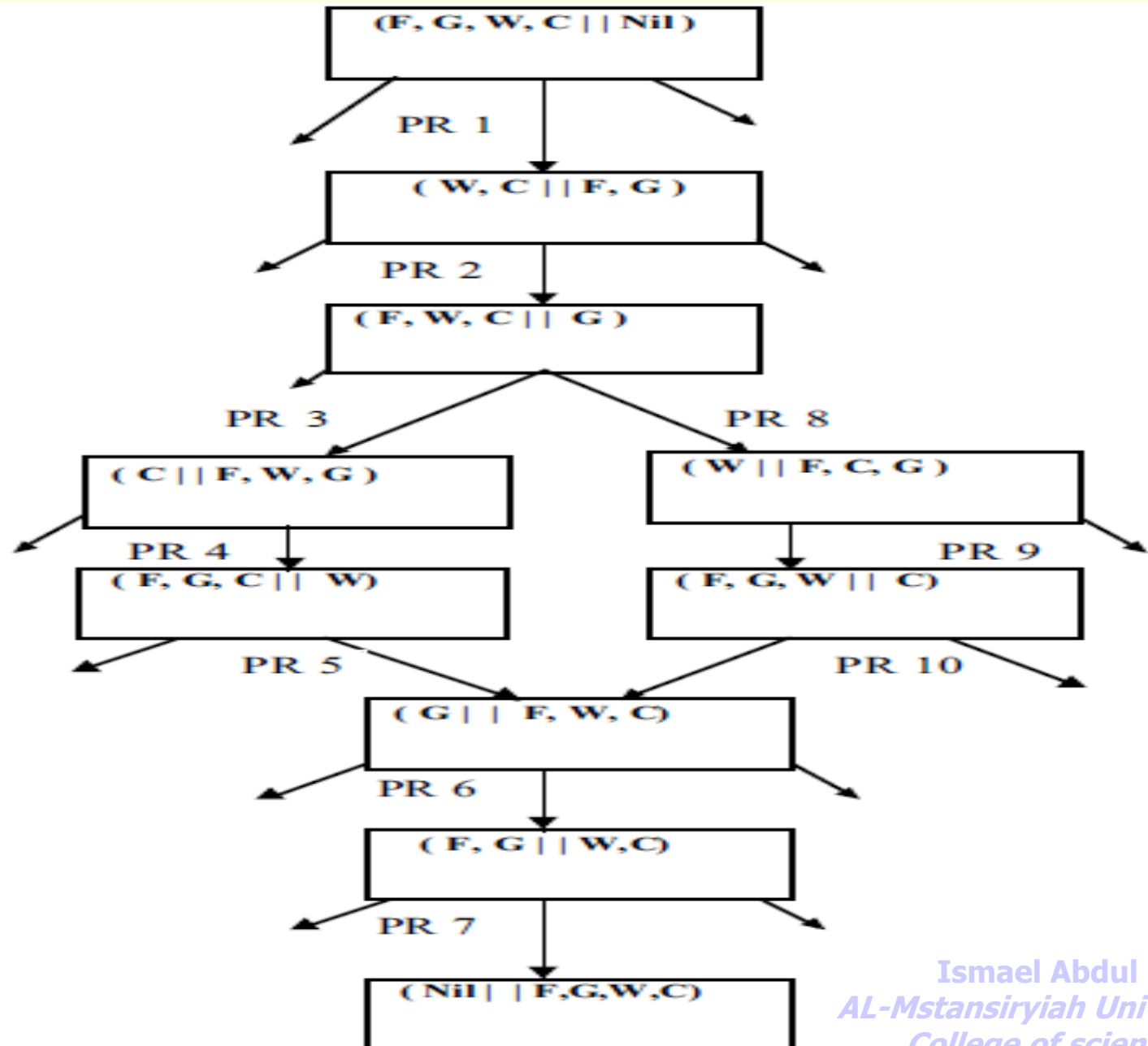
The illegal states in the problem are $(W, G \mid \mid F, C)$, $(G, C \mid \mid F, W)$, $(F, W \mid \mid G, C)$ and $(F, C \mid \mid W, G)$ where F, G, $\mid \mid$, W and C denote the farmer, the goat, the river, the wolf and the cabbage respectively. In the first case the wolf and the goat are at the **left bank**, and the farmer bank of the river. The cabbage are at the right and the second case demonstrates the presence of goat and cabbage in the left and the farmer and the wolf in the the other illegal states can be **right bank**. Similarly, explained easily.

knowledge base

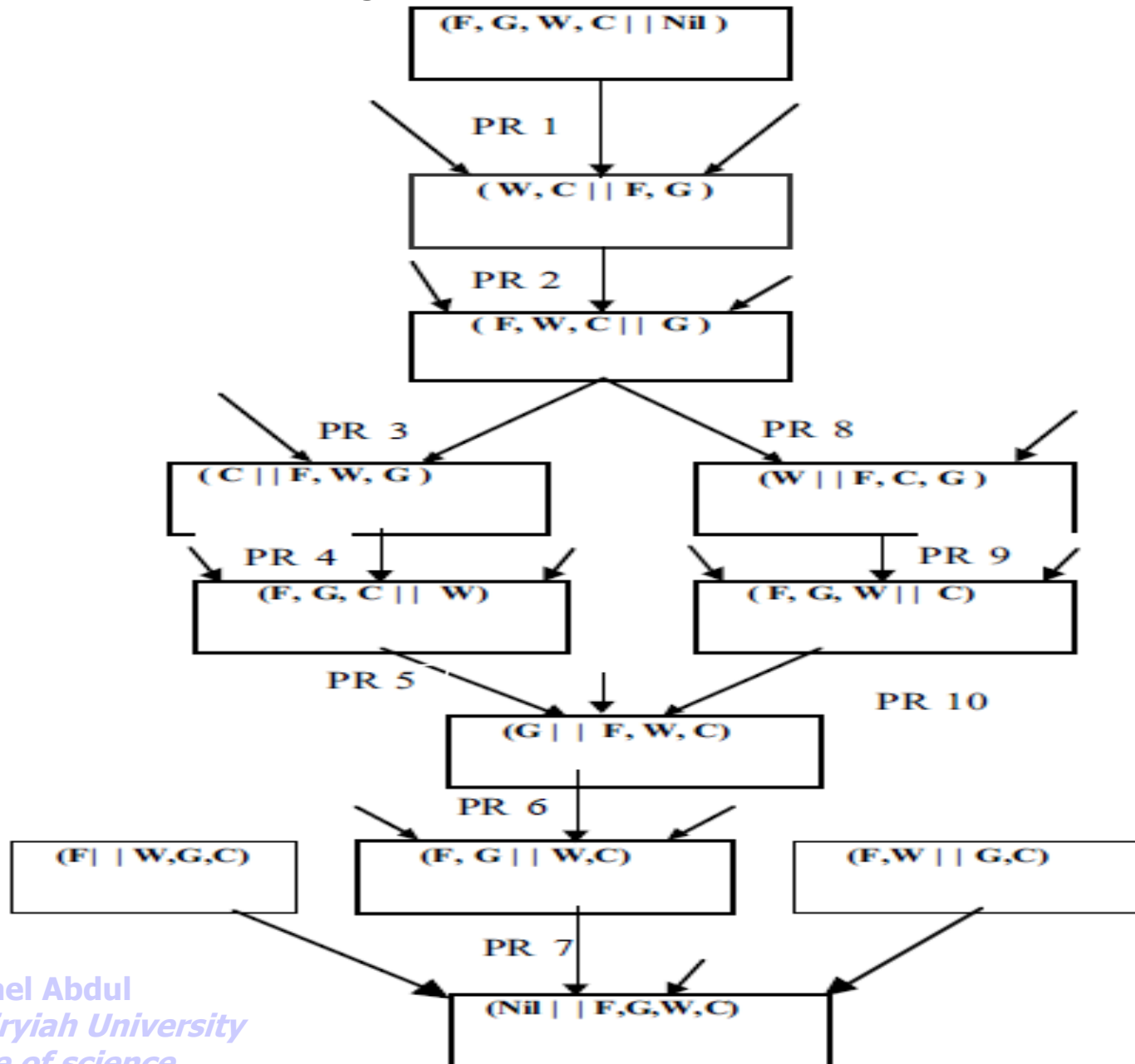
- PR 1: (F, G, W, C || Nil) \rightarrow (W, C || F, G)
PR 2: (W, C || F, G) \rightarrow (F, W, C || G)
PR 3: (F, W, C || G) \rightarrow (C || F, W, G)
PR 4: (C || F, W, G) \rightarrow (F, G, C || W)
PR5: (F, G, C || W) \rightarrow (G || F, W, C)
PR 6: (G || F, W, C) \rightarrow (F, G || W, C)
PR 7: (F, G, || W, C) \rightarrow (Nil || F,G, W, C)
PR 8 (F, W, C || G) \rightarrow (W || F, G, C)
PR 9: (W || F, G, C) \rightarrow (F, G, W || C)
PR 10: (F, G, W || C) \rightarrow (G || F, W, C)
PR 11: (G || F, W, C) \rightarrow (F, G || W,C)
PR 12: (F, G || W, C) \rightarrow (Nil || F, G, W, C)

Forward Reasoning: Given the starting state (F, G, W, C | | Nil) and the goal state (Nil | | F, G, W, C), one may expand the state-space, starting with (F,G,W,C | | Nil) by the supplied knowledge base, as follows:

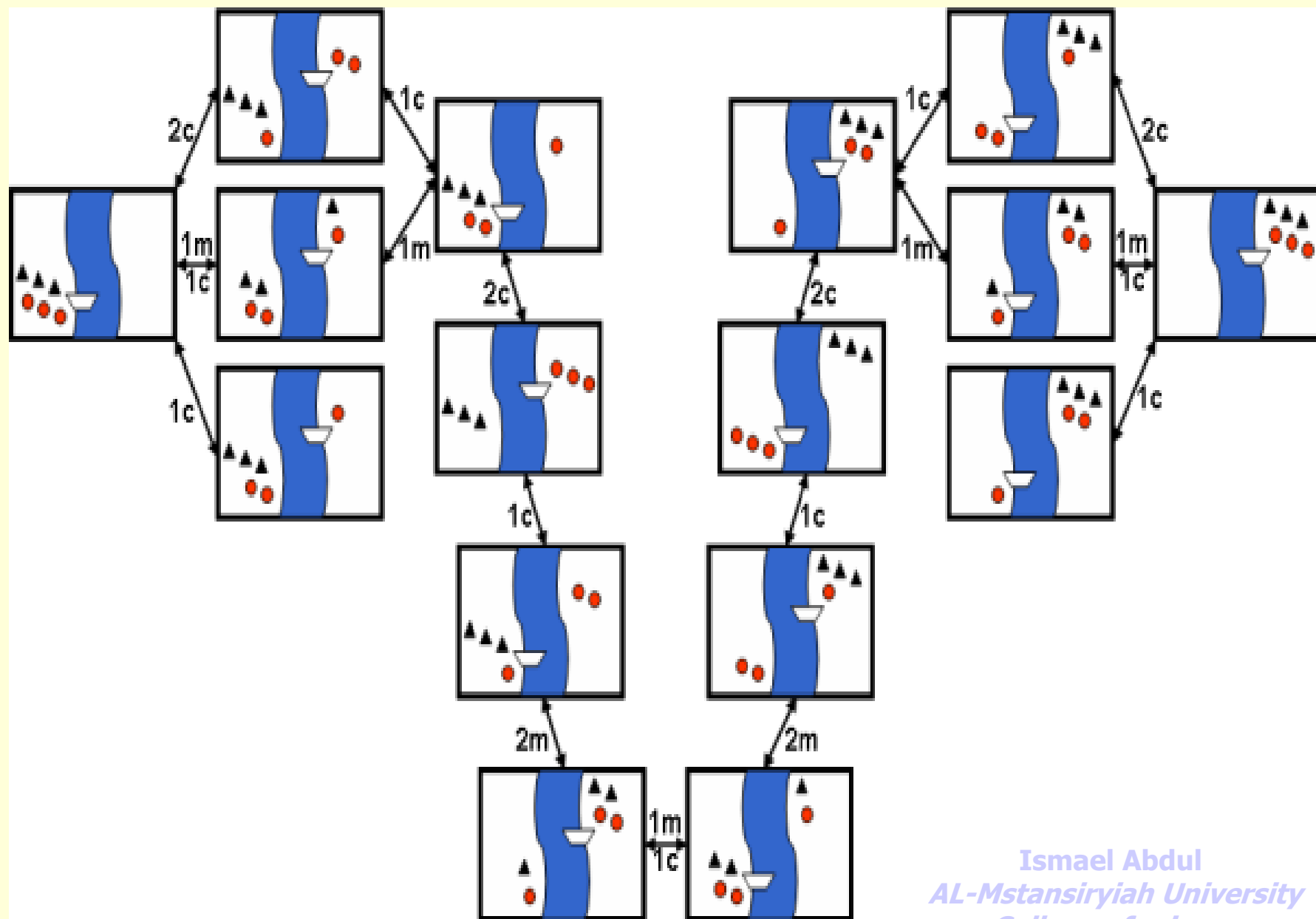
The forward reasoning trace of the farmer's problem with a partially expanded state-space.



Backward reasoning solution of the farmer's problem.



On one bank of a river are *three missionaries* and *three cannibals*. There is one boat available that can hold up to two people and that they would like to use to cross the river. If the cannibals ever outnumber the missionaries on either of the river's banks, the missionaries will get eaten. How can the boat be used to safely carry all the missionaries and cannibals across the river ?



RN	Left side of rule	→	Right side of rule
<i>Rules for boat going from left bank to right bank of the river</i>			
L1	$[(n_1M, m_1C, 1B), [n_2M, m_2C, 0B)]$	→	$[(n_1-2)M, m_1C, 0B], [(n_2+2)M, m_2C, 1B]$
L2	$[(n_1M, m_1C, 1B), [n_2M, m_2C, 0B)]$	→	$[(n_1-1)M, (m_1-1)C, 0B], [(n_2+1)M, (m_2+1)C, 1B]$
L3	$[(n_1M, m_1C, 1B), [n_2M, m_2C, 0B)]$	→	$[(n_1M, (m_1-2)C, 0B], [n_2M, (m_2+2)C, 1B]$
L4	$[(n_1M, m_1C, 1B), [n_2M, m_2C, 0B)]$	→	$[(n_1-1)M, m_1C, 0B], [(n_2+1)M, m_2C, 1B]$
L5	$[(n_1M, m_1C, 1B), [n_2M, m_2C, 0B)]$	→	$[(n_1M, (m_1-1)C, 0B], [n_2M, (m_2+1)C, 1B]$
<i>Rules for boat coming from right bank to left bank of the river</i>			
R1	$[(n_1M, m_1C, 0B], [n_2M, m_2C, 1B)]$	→	$[(n_1+2)M, m_1C, 1B], [(n_2-2)M, m_2C, 0B]$
R2	$[(n_1M, m_1C, 0B], [n_2M, m_2C, 1B)]$	→	$[(n_1+1)M, (m_1+1)C, 1B], [(n_2-1)M, (m_2-1)C, 0B]$
R3	$[(n_1M, m_1C, 0B], [n_2M, m_2C, 1B)]$	→	$[(n_1M, (m_1+2)C, 1B], [n_2M, (m_2-2)C, 0B]$
R4	$[(n_1M, m_1C, 0B], [n_2M, m_2C, 1B)]$	→	$[(n_1+1)M, m_1C, 1B], [(n_2-1)M, m_2C, 0B]$
R5	$[(n_1M, m_1C, 0B], [n_2M, m_2C, 1B)]$	→	$[(n_1M, (m_1+1)C, 1B], [n_2M, (m_2-1)C, 0B]$

TABLE 2.13 Solution Path

Rule number	$[(3M, 3C, 1B), (0M, 0C, 0B)] \leftarrow \text{Start State}$
L2:	$[(2M, 2C, 0B), (1M, 1C, 1B)]$
R4:	$[(3M, 2C, 1B), (0M, 1C, 0B)]$
L3:	$[(3M, 0C, 0B), (0M, 3C, 1B)]$
R5:	$[(3M, 1C, 1B), (0M, 2C, 0B)]$
L1:	$[(1M, 1C, 0B), (2M, 2C, 1B)]$
R2:	$[(2M, 2C, 1B), (1M, 1C, 0B)]$
L1:	$[(0M, 2C, 0B), (3M, 1C, 1B)]$
R5:	$[(0M, 3C, 1B), (3M, 0C, 0B)]$
L3:	$[(0M, 1C, 0B), (3M, 2C, 1B)]$
R5:	$[(0M, 2C, 1B), (3M, 1C, 0B)]$
L3:	$[(0M, 0C, 0B), (3M, 3C, 1B)] \rightarrow \text{Goal state}$

Home work

1. *Find another solution to the water jug problem
(4, 3) jugs.*
2. *Draw the state space search of the water jug problem
(4, 3) jug and (5, 3) jugs*
3. *Draw missionaries cannibals state space search*
4. *Suggest another way to solve water jug problems*