

$$\frac{1}{y - ay^2} = \frac{A - aAy + By}{y - ay^2}$$

$$1 = A + (-aA + B)y$$

$$A = 1$$

$$-aA + B = 0 \Rightarrow -a + B = 0 \Rightarrow B = a$$

$$\Rightarrow \frac{1}{y - ay^2} = \frac{1}{y} + \frac{a}{1 - ay}$$

$$\Rightarrow \int \left( \frac{1}{y} + \frac{a}{1 - ay} \right) dy = \int \frac{1}{a+x} dx$$

$$\Rightarrow \ln y - \ln(1 - ay) = \ln(a+x) + c$$

$$\ln \frac{y}{(1 - ay)(a+x)} = \ln e^c$$

$$\Rightarrow y = (1 - ay)(a+x)e^c$$

(f)  $3e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$

$$3e^x \tan y \, dx = -(1 - e^x) \sec^2 y \, dy$$

$$\frac{\sec^2 y}{\tan y} \, dy = -\frac{3e^x}{1 - e^x} \, dx$$

$$\int \frac{\sec^2 y}{\tan y} \, dy = -3 \int \frac{e^x}{1 - e^x} \, dx$$

$$\ln \tan y = 3 \ln(1 - e^x) + \ln c$$

$$\ln \tan y = \ln(1 - e^x)^3 + \ln c$$

$$\Rightarrow \ln \frac{\tan y}{(1 - e^x)^3} = \ln c$$