$\boxed{1}$ To find the equation $\boxed{T\alpha^{\gamma-1} = \text{Const.}}$ for Poisson relation, we start with the first form of the first law of thermodynamic for ideal gases: $C_v\, dT = dq_w - p\, d\alpha$

- for adiabatic process, $dq_w = 0$, then the equation will be $C_v\, dT = -p\, d\alpha$ --- (1)

$\because p\alpha = R'T \longrightarrow P = R'\frac{T}{\alpha}$ --- (2)

put (2) in (1) and arranging the equation

$C_v\, dT \cancel{+} -P\, d\alpha = \cancel{\text{zero}} \Longrightarrow C_v\, dT = -(R'\frac{T}{\alpha})\, d\alpha$

$\Longrightarrow C_v\, dT = -R'T\frac{d\alpha}{\alpha} \Longrightarrow C_v\frac{dT}{T} = -R'\frac{d\alpha}{\alpha}$

$\Longrightarrow C_v\frac{dT}{T} + R'\frac{d\alpha}{\alpha} = 0$ ثم تكامل المعادلة

$C_v \ln T + R' \ln\alpha = \text{Const.}$ ثم نقسم على C_v

$\ln T + \frac{R'}{C_v}\ln\alpha = \frac{\text{Const}}{C_v} = \text{Const.}$

حسب القاعدة الرياضية فإن

$x \ln y = \ln y^x$

$\Longrightarrow \frac{R'}{C_v}\ln\alpha = \ln\alpha^{\frac{R'}{C_v}}$

$\therefore \ln T + \ln\alpha^{R'/C_v} = \text{Const.}$

حسب القاعدة الرياضية

$\ln x + \ln y = \ln(xy)$

$\therefore \ln(T\alpha^{R'/C_v}) = \text{Const.}$

حسب القاعدة الرياضية نأخذ e للمعادلة ينتج

$e^{\ln x} = x$

$T\alpha^{R'/C_v} = \text{Const.} \Longrightarrow T\alpha^{\frac{C_p - C_v}{C_v}} = \text{Const.}$ Poisson $\boxed{1}$

$\because C_p - C_v = R'$

$\gamma = \frac{C_p}{C_v} - \frac{C_v}{C_v}$

$\Longrightarrow T\alpha = \text{Const} \Longrightarrow \boxed{T\alpha^{\gamma-1} = \text{Const}}$

المعادلة الأولى من صيغة بويزن

$\boxed{2}$ To find the equation $\boxed{P\alpha^{\gamma} = const.}$ for Poisson relation, we start with the equation $T\alpha^{\gamma-1} = const$ --- (1), and using the ideal gas law $P\alpha = R'T \Longrightarrow T = \frac{P\alpha}{R'}$ --- (2)

Put (2) in (1)

$$\frac{P\alpha}{R'}\,\alpha^{\gamma-1} = Const.$$

$$P\alpha\,\alpha^{\gamma-1} = const.\,R' = Const.$$

$$P\alpha^{1+\gamma-1} = Const.$$

Poisson $\boxed{2}$

المعادلة الثانية
من صيغة بويزن

$$\Longrightarrow \boxed{P\alpha^{\gamma} = Const}$$

3 To find the equation $\boxed{T P^{(1-\gamma)/\gamma} = \text{Constant}}$ for Poisson relation, we start with the second form of the first law of thermodynamic for ideal gases

$$c_p\, dT = dq + \alpha\, dP$$

- for adiabatic process, $dq = 0$, then

$$c_p\, dT = \alpha\, dP \quad --- \quad (1)$$

$$\because P\alpha = R' T \longrightarrow \alpha = R' \frac{T}{P} \quad -- \quad (2)$$

Put (2) in (1)

$$c_p\, dT = R' T \frac{dP}{P} \Longrightarrow c_p \frac{dT}{T} = R' \frac{dP}{P}$$

$$\Longrightarrow c_p \frac{dT}{T} - R' \frac{dP}{P} = \text{Zero}$$ ثم نكامل المعادلة

$$c_p \ln T - R' \ln P = \text{Const.}$$

ثم نقسم على c_p

$$\ln T - \frac{R'}{c_p} \ln P = \frac{\text{Const}}{c_p} = \text{Const.}$$

حسب القاعدة الرياضية

$$x \ln y = \ln y^x$$

$$\therefore -\frac{R'}{c_p} \ln P = \ln P^{-R'/c_p} \Longrightarrow \ln T + (\ln P^{-R'/c_p}) = \text{Constan}$$

حسب القاعدة الرياضية

$$\ln x + \ln y = \ln (xy)$$

$$\therefore \ln T + (\ln P^{-R'/c_p}) = \text{Const} \Longrightarrow \ln (T P^{-R'/c_p}) = \text{Con}$$

حسب القاعدة الرياضية

$$e^{\ln x} = x$$

ناخذ e للمعادلة ينتج

$$T P^{-R'/c_p} = \text{Const.} \qquad \because c_p - c_v = R'$$

$$\Longrightarrow T P^{-(c_p - c_v)/c_p} = \text{Const} \Longrightarrow T P^{\frac{-c_p + c_v}{c_p}} = \text{Const}$$

$$\gamma = \frac{c_p}{c_v}$$

$$\Longrightarrow T P^{-\frac{c_p}{c_p} + \frac{c_v}{c_p}} = \text{Const} \Longrightarrow T P^{-1 + \frac{1}{\gamma}} = \text{Const}$$

$$\Longrightarrow T P^{\frac{-\gamma + 1}{\gamma}} = \text{Const} \Longrightarrow \boxed{T P^{1-\gamma/\gamma} = \text{Const}}$$

المعادلة الثالثة من صيغ بويزن Poisson 3

4 For an isobaric processes, show that

(a) $\Delta u = c_p (T_f - T_i) + P(\alpha_i - \alpha_f)$

الحل

$\therefore h = u + P\alpha$ قانون الإنثالبي

$\therefore dh = du + P\,d\alpha + \alpha\, dP$

For isobaric processes, P is constant $\rightarrow dP = $ Zero

$\therefore dh = du + P\,d\alpha \Longrightarrow du = dh - P\,d\alpha$

$\therefore \boxed{dh = c_p\, dT} \longleftarrow$ (for ideal gas)

$\Longrightarrow du = c_p\, dT - P\, d\alpha$

تكامل طرفي المعادلة

$$\int du = \int_{T_i}^{T_f} c_p\, dT - \int_{\alpha_i}^{\alpha_f} P\, d\alpha \Longrightarrow$$

$$\Delta u = c_p (T_f - T_i) - P(\alpha_f - \alpha_i)$$

$$\Longrightarrow \boxed{\Delta u = c_p (T_f - T_i) + P(\alpha_i - \alpha_f)}$$

(b) Is this true for all gasses? Or only ideal gasses?

الحل For ideal gasses only because we used the equation $\boxed{dh = c_p\, dT}$ which is only used for ideal gases.

5. Show that $\theta = T\left(\frac{P_o}{P}\right)^{R_d/c_p}$, by using the following form of the first law of thermodynamic for ideal gas law, $c_p\,dT = dq + \alpha\,dP$, where $dq = \text{Zero}$

الحل

$c_p\,dT = dq + \alpha\,dP$ (for adiabatic proccess), then

$\therefore dq = 0$

$\therefore c_p\,dT - \alpha\,dP = 0$ --- (1)

$\therefore P\alpha = RT$ --- (2) $\Longrightarrow \alpha = \frac{RT}{P}$ --- (3)

put (3) in (1)

ثم نغير ترتيب المعادلة

$$c_p\,dT - \frac{RT}{P}\,dP = 0$$

$$\frac{dT}{T} = \frac{R}{c_p}\,\frac{dP}{P}$$

نكامل المعادلة لعتم ضغط من $(P_o \rightarrow P)$
ويتم درجة حرارة من $(\theta \rightarrow T)$

$$\int_{\theta}^{T}\frac{dT}{T} = \int_{P_o}^{P}\frac{R}{c_p}\,\frac{dP}{P}$$

$$\ln T\Big|_{\theta}^{T} = \frac{R}{c_p}\ln P\Big|_{P_o}^{P} \quad \text{--- (4)}$$

$$\frac{R}{c_p}(\ln P - \ln P_o) \Longrightarrow \frac{R}{c_p}\ln\frac{P}{P_o} \Longrightarrow \left(\ln\frac{P}{P_o}\right)^{R/c_p}$$

$$\ln T - \ln\theta \Longrightarrow \ln\left(\frac{T}{\theta}\right)$$

$\therefore$ equation 4 become:

$$\ln\left(\frac{T}{\theta}\right) = \ln\left(\frac{P}{P_o}\right)^{R/c_p}$$

نأخذ exp للطرفين

$$\frac{T}{\theta} = \left(\frac{P}{P_o}\right)^{R/c_p} \Longrightarrow T = \theta\left(\frac{P}{P_o}\right)^{R/c_p}$$