(f) $\sec x + \tan y \frac{dy}{dx} = 0$

الحل :

$$\tan y \frac{dy}{dx} = -\sec x$$

$$\tan y \, dy = -\sec x \, dx$$

$$\int \tan y \, dy = -\int \sec x \, dx$$

$$-\ln \cos y = -\ln |\sec x + \tan x| + \ln c$$

$$\ln |\sec x + \tan x| - \ln \cos y = \ln c$$

$$\Rightarrow \ln \frac{\sec x + \tan x}{\cos y} = \ln c$$

$$\Rightarrow \frac{\sec x + \tan x}{\cos y} = c$$

$$\Rightarrow \sec y [\sec x + \tan x] = c$$

(g) $\frac{1}{r} \frac{dr}{d\varnothing} = \cot \varnothing$

الحل :

$$\frac{1}{r} dr = \cot \varnothing \, d\varnothing$$

$$\int \frac{1}{r} dr = \int \cot \varnothing \, d\varnothing$$

$$\Rightarrow \ln r = \ln (\sin \varnothing) + \ln c$$

$$\Rightarrow \ln \left(\frac{r}{\sin \varnothing} \right) = \ln c$$

$$\Rightarrow \frac{r}{\sin \varnothing} = c \quad \Rightarrow \quad r = c \sin \varnothing$$