

12) Solve $(2y - x - 4) dx = (2x - y + 2) dy$

$$\begin{aligned} 2y - x - 4 &= 0 \\ -y + 2x + 2 &= 0 \cdot 2 \end{aligned}$$

$$\begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} = 1 - 4 = -3 \neq 0$$

$$\begin{aligned} 2y - x - 4 &= 0 \\ -2y + 4x + 4 &= 0 \end{aligned}$$

$$3x = 0 \rightarrow x = 0 \rightarrow y = 2$$

$$(h, k) = (0, 2)$$

$$x = x_1 + h, y = y_1 + k, x = x_1 \Rightarrow dx = dx_1$$

$$y = y_1 + 2 \rightarrow dy = dy_1$$

$$y_1 = v x_1$$

$$dy_1 = v dx_1 + x_1 dv$$

$$[2(y_1 + 2) - x_1 - 4] dx_1 = [2x_1 - (y_1 + 2)] dy_1$$

$$(2y_1 + 4 - x_1 - 4) dx_1 = (2x_1 - y_1 - 2 + 2) dy_1$$

$$(2v x_1 - x_1) dx_1 = (2x_1 - v x_1) (v dx_1 + x_1 dv)$$

$$(2v x_1 - x_1 - 2x_1 v + v^2 x_1) dx_1 - (2x_1^2 - x_1^2 v) dv = 0$$

$$x_1(v^2 - 1) dx_1 - x_1^2(2 - v) dv = 0$$

$$\int \frac{dx_1}{x_1} - \int \frac{(2-v)}{v^2-1} dv = 0$$

$$\ln|x_1| + \left(\int \frac{1}{v-1} dv - \frac{3}{2} \int \frac{1}{v+1} dv \right) = \ln C$$

$$\ln|x_1| - \frac{1}{2} \ln|v-1| + \frac{3}{2} \ln|v+1| = \ln C$$

$$\ln|x| - \frac{1}{2} \ln\left|\frac{y-2}{x}-1\right| + \frac{3}{2} \ln\left|\frac{y-2}{x}+1\right| = \ln C$$

$$\frac{2-v}{v^2-1} = \frac{2-v}{(v-1)(v+1)} = \frac{A}{v-1} + \frac{B}{v+1}$$

$$\rightarrow 2-v = A(v+1) + B(v-1)$$

$$2-v = Av + A + Bv - B$$

$$2-v = (A+B)v + (A-B)$$

$$A+B = -1, A-B = 2$$

$$A = 2+B$$

$$2+B+B = -1 \rightarrow 2+2B = -1$$

$$\rightarrow 2B = -3 \Rightarrow B = -\frac{3}{2}$$