

11 Construct a group in terms of generators & relations

Example: specify a set of elements called the generators such that every element of the group can be written as a product of these generators and their powers.

List of equations, called the relations that completely specify the group operation, in the sense that the entire table is determined by those equations. For example

$$D_n = \langle x, y : x^n = 1, y^2 = 1, yx = x^{-1}y \rangle$$

called dihedral group of order $2n$.

Remark: $x^n = 1$ and $y^2 = 1$ can be stated as $x^n = y^2 = 1$

and $(xy)^2 = 1$ instead of the relation $yx = x^{-1}y$

$$Z_K = \langle x : x^K = 1 \rangle$$

$$V_4 = \langle x, y : x^2 = y^2 = 1, yx = xy \rangle$$

such expressions called presentation of the group in terms of generators and relations.