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Example: (1) Let $G = \langle x \rangle$ a cyclic group of order 4, thus $G = \{1, x, x^2, x^3\}$ and define $T: G \to GL(3, \mathbb{R})$ by

$T(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$, note that $T(x)^4 = I_3$ where $x^4 = 1$

to check that T is a homomorphism we have to prove:

$T(1 * x) = T(1) \cdot T(x)$, $T(1 * x^2) = T(1) \cdot T(x^2)$

$T(1 * x^3) = T(1) \cdot T(x)^3$ check!

$T(x * x^2) = T(x) \cdot T(x)^2$, $T(x * x^3) = T(x) \cdot T(x)^3$

$T(x^2 * x^3) = T(x)^2 \cdot T(x)^3$. and so T is a representation of degree 3.

(2)

Define $U: G \to GL(2, \mathbb{R})$ by $U(x) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

note that $U(x)^4 = I_2$, $U(x^2) = U(1) = I_2$ and $U(x^3) = U(x)$, U is a representation of G of degree 2.