Group Representation

Def.: Let a representation of a group G is a homomorphism $T: G \to G'$ where G' is a subgroup of the general linear group $GL(n, \mathbb{C})$, n is called the degree or dimension of the representation T.

Examples: Let $G = \langle x \rangle$ of order 2 i.e $G = \{1, x\}$

define $T: G \to GL(n, \mathbb{R})$ by $T(x) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $T(1) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$U: G \to GL(n, \mathbb{R})$ by $U(x) = [-1]$, $U(1) = [1]$

T is a representation of G of degree 2

U is a representation of G of degree 1. (Check!)

Remarks: (1) if G has 1 as identity and T is a representation of G then $T(1) = I_n$ (identity matrix of size $n \times n$ where n is the degree of T).

- if $g \in G$, $T(g^{-1}) = T(g)^{-1}$, $T(g^k) = T(g)^k$ for any integer k

- We need only give the values of T on a set of generators