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(1) show that for $n=1$, $GL(n,\mathbb{R}) \trianglelefteq GL(n,\mathbb{C})$ and for $n=2$, $GL(n,\mathbb{R}) \ntrianglelefteq GL(n,\mathbb{C})$.

Hint: Take $X=\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ and $Y=\begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix}$ and consider $Y^{-1}XY$.

(2) Let $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

show that $\langle A \rangle$ is a cyclic group of order 3 and $\langle B \rangle$ is a cyclic group of order 6.

(3): prove that $SL(n,\mathbb{C}) \trianglelefteq GL(n,\mathbb{C})$ & $SL(n,\mathbb{R}) \trianglelefteq GL(n,\mathbb{C})$

Hint: use determinants.

(4): Let A as in (2), $C=(-1)I_3$, show that $C^2=I_3$ and $AC=CA$, such that $\langle A, C \rangle$ is cyclic of order 6

(5): prove that $(GL(n,\mathbb{C}), \cdot)$ is a group where $\cdot$ is the usual multiplication of matrices.