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Def. Let $\phi: G \rightarrow G'$ be a homomorphism, define
kernel of ϕ by: $\text{Ker}(\phi) = \{x \in G \mid \phi(x) = e'\}$
where e' is the identity of the group G' .

Remark - $e \in \text{Ker}(\phi)$ since $\phi(e) = e'$.

— $\text{Ker}(\phi) = g^{-1} \times \text{Ker}(\phi) \times g, \forall g \in G$, where $(G, \times)$ is
the group of the domain of ϕ . (CHECK!)

Def. Let $N < G$ then N is called normal subgroup
if $g^{-1}Ng = N, \forall g \in G$ and denoted by $N \trianglelefteq G$

— if N is proper normal we write $N \triangleleft G$

— example $\{e\} \triangleleft G$ for any group G . and $G \triangleleft G$

— $\text{Ker}(\phi) \triangleleft G$

Def. The center of a group G denoted by $C(G)$
is the set $C(G) = \{c \in G \mid c \times x = x \times c, \forall x \in G\}$

— $C(G) \triangleleft G$ since, if $g \in G$ and $x \in G$ we have

$$x^{-1}gx = x^{-1}xg = e \cdot g = g \in C(G).$$