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Def: Let $(G, \cdot)$ be a group, $H \subseteq G$, if H is itself a group i.e. $(H, \cdot)$ is a group, we say that

H is a subgroup of G and write as $H \leq G$

if H is a proper subset of G , we say H is a proper subgroup of G and write as: $H < G$.

proposition: Let $H \leq G$ then $H \leq G \iff xy^{-1} \in H$

$\forall x, y \in H$.

Example: If G act on Ω then the the stabilizer G_x for $x \in \Omega$ is a subgroup of G i.e. $G_x \leq G$.

Remark: $(xy)^{-1} = y^{-1}x^{-1} \quad \forall x, y \in G$.

Def: (Homomorphism), Let G, G' be two groups a function ϕ from G to G' is called a homomorphism if $\phi: G \rightarrow G'$, $\phi(gh) = \phi(g)\phi(h) \quad \forall g, h \in G$

Remark: (1) $\phi(e) = e'$ where e and e' are the identities of G, G' respectively.

(2) $\phi(g^{-1}) = \phi(g)^{-1} \quad \forall g \in G$.