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3 - cyclic groups: $\langle g \rangle = G = \mathbb{Z}_k$

where $\mathbb{Z}_k = \{e, g, g^2, \dots, g^{k-1}\}$, where multiplication

defined by $g^i \cdot g^j = g^{i+j}$ and $g^0 = e = g^k$

Note if $i+j > k-1$ say $i+j = k+m$ then $0 \leq m < k$

then $g^{i+j} = g^{k+m} = g^k \cdot g^m = e g^m = g^m$

for example $\mathbb{Z}_2 = \{e, g\}$ & $g^2 = e$

$\mathbb{Z}_3$ have the following multiplication table

	e	g	g ²
e	e	g	g ²
g	g	g ²	e
g ²	g ²	e	g

Remarks: 1 - for any group G ,

$$(x^{-1})^{-1} = x \quad \forall x \in G.$$

2 - The number of elements of G is called the order of the group denoted by $|G|$,

it may be finite as examples 2 and 3 or infinite

as example 1.

3 - If $xy = yx \quad \forall x, y \in G$, we say that

G is abelian or commutative.