

1

Representation theory

Def. A pair $(G, \cdot)$ of a non-empty set G and binary operation $(\cdot)$ [multiplication] is called a group if it satisfies the following:

- (1) $x \cdot y \in G \quad \forall x, y \in G$ (closure)
- (2) $(x \cdot y) \cdot z = x \cdot (y \cdot z) \quad \forall x, y, z \in G$ (associative)
- (3) $\exists!$ (there exists a unique) $e \in G$ such that $x \cdot e = e \cdot x = x \quad \forall x \in G$. (e is called identity)
- (4) $\forall x \in G, \exists! x^{-1} \in G$ such that $x \cdot x^{-1} = x^{-1} \cdot x = e$ (x^{-1} called inverse of x)

Examples: 1- $(\mathbb{R}, +)$, $(\mathbb{R}, \cdot)$, $(\mathbb{Z}, +)$, $(\mathbb{Z}, \cdot)$ form a groups (check!).

2- $G = \{e, a, b, ab\}$ with multiplication table

	e	a	b	ab
e	e	a	b	ab
a	a	e	ab	b
b	b	ab	e	a
ab	ab	b	a	e

this group usually denote by V_4 and called Klein 4-group