

# Permutation and Combination

(تباديل وتوافيق)

## 1- Permutation

(بكتش)

a. Permutation (with out repetition) التباديل بدون تكرار

It's arrangement of different objects in different forms, by using the following formula

هي ترتيب أشياء مختلفة بصفات مختلفة

$$P_r^n = \frac{n!}{(n-r)!}, r \leq n$$

such that (s.t.)

$P$ : Permutation  
 $r$ : different objects

$r$ : taken objects at time <sup>في المرة الواحدة منها</sup>

There are two cases:-

- The  $n$ -objects are different.

The objects are repeated more than once.

E) How many ways can be arranged three different books & eight books.

$$n = 8, r = 3$$

$$\therefore \text{of ways} = P_r^n = P_3^8 = \frac{8!}{(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!}} = 336$$

|   |   |   |
|---|---|---|
| 8 | 7 | 6 |
|---|---|---|

EX(2) How many numbers (consist of 4-digits) can be formed from the digits: 1, 2, 4, 5, 7, 8.

عدد الأرقام (عدد مكون من 4-مئات) التي يمكن تشكيلها من الأرقام 1، 2، 4، 5، 7، 8.

Sol  $n = 6, r = 4,$

$$\therefore \text{no. of numbers} = P_4^6 = \frac{6!}{(6-4)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2!} = 360$$

another Sol: 

|   |   |   |   |
|---|---|---|---|
| 6 | 5 | 4 | 3 |
|---|---|---|---|

 $= 6 \cdot 5 \cdot 4 \cdot 3 = 360$  numbers.

H.W.  
EX(3)

How many words (consist of 3 letters) can be formed from the letters: NMORZ.

Sol  $n = 5, r = 3$

$$\therefore \text{no of words} = P_3^5 = \frac{5!}{2!} = 60$$

H.W.  
EX(4)

How many different four letters words can be formed from the letters of the word: MANLY.

Sol  $P_4^4 = 4! = 24$

H.W.  
Q

Prove that  $P_n^n = n!$

Proof  $P_n^n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! = n(n-1)(n-2) \dots$

EX(5) How many different five letters words can be formed from the letters of the word: COMBINATION

Sol  $n = 8, r = 5, \therefore \text{no. of words} = P_5^8 = 6720$  words.

### (3) دُستَن (Permutation (with repetition))

It's arrangement of different objects ( $n$ ), taken ( $r$ ) at each once, by using the following formula:

هي ترتيب  $n$  من الأشياء المختلفة مأخوذة مرة واحدة باستخدام الصيغة التالية

$$\text{no. of arrang.} = n^r$$

Ex: How many numbers (consist of five digits) can be formed from the digits: 1, 2, 3, 4, 6, 8, 9 (with repetition)

Sol(1)  $n = 7, r = 5$

$$\therefore \text{no. of numbers} = 7^5 = 16807$$

Sol(2):  $\boxed{7} \boxed{7} \boxed{7} \boxed{7} \boxed{7} = 7^5 = 16807 \text{ words}$

### Some Rules In Permutation

Rule(1):  $\text{no. of arrang.} = \frac{n!}{r_1! r_2! r_3! \dots r_k!}$

s.t.:

$r_1$ : the first kind

$r_2$ : the second kind

!

$r_k$ : the  $k$ .th kind

$$r_1 + r_2 + r_3 + \dots + r_k = n$$

وهذا يعني: عدد الطرق (الترتيبات) المختلفة لترتيب ( $n$ ) من الأشياء المختلفة مأخوذة جميعها في كل مرة، إذا كانت  $r_1$  من تلك الأشياء متماثلة ومن نوع واحد،  $r_2$  منها متماثلة ومن نوع  $r_2$ ،  $r_3$  منها متماثلة ومن نوع ثالث، ...،  $r_k$  منها متماثلة ومن نوع  $k$ . (نطبق القانون أعلاه).