2- Combination سؤال ١٧٨

It's a selection of different objects (r), taken from a set of different objects (n) without required to the arrangement of them. it's given by:-

$$C_r^n = \frac{n!}{r!\,(n-r)!}, \quad r \le n$$

$$= \frac{P_r^n}{r!}, \quad r \le n$$

Ex(1) How manys can be selected (chosen) comittee of four persons from six?

Sol $n = 6$, $r = 4$

$$\therefore C_r^n = C_4^6 = \frac{6!}{4!\,(6-4)!} = \frac{6.5.\cancel{4!}}{\cancel{4!}\;2!} = 15 \text{ no. of selected}$$

Ex(2) we want to select comitte of six members from five teachers and ten students; find:-

1) How many ways can be selected?
2) How many ways can be selected s.t.: no. of teachers always more than the students?
3) There are at most 2 students?

Sol 1) $n = 5 + 10 = 15$, $r = 6$, $\therefore C_6^{15} = \frac{15!}{6!\,(15-6)!} = 5005$ no. of selec.

2) no. of selec. $= C_5^5 C_1^{10} + C_4^5 C_2^{10} = 10 + 225 = 235$

3) no. of selec. $= C_1^{10} C_5^5 + C_2^{10} C_4^5 = 235$

if he want

H.w. 4) There are at least 2 students:-

no. of selec. = $C^{10}_2 \, C^5_4 + C^{10}_3 \, C^5_3 + C^{10}_4 \, C^5_2 + C^{10}_5 \cdot C^5_1 = \square$

5) no. of students are always more than the teachers?

H.w. no. of selec. = $C^{10}_5 \, C^5_1 + C^{10}_4 \, C^5_2 = + C^{10}_6 \, C^5_0 = \square$

Q) Find the value of n if :-

① $C^{n+1}_2 = 3$;

notes:

1) $C^n_n = 1$ H.w.

2) $C^n_1 = n$

3) $C^n_0 = 1$

~~Some useful mathematical Formulas~~

IF (n) is a positive integer number, then :-

$$\boxed{(a+b)^n = \sum_{r=0}^{n} C_r^n \, a^r \, b^{n-r}} \quad \text{Binomial's Law}$$

$$= \underbrace{C_0^n}_{1} \underbrace{a^0}_{1} b^{n-0} + C_1^n a^1 b^{n-1} + C_2^n a^2 b^{n-2} + \dots + \underbrace{C_n^n}_{1} a^n \underbrace{b^{n-n}}_{1}$$

$$= b^n + n\,a\,b^{n-1} + \frac{n(n-1)}{2!} a^2 b^{n-2} + \frac{n(n-1)(n-2)}{3!} a^3 b^{n-3} + \dots a^n$$

EX(1): if $n=2$, Find $(a+b)^n$

$$(a+b)^2 = \sum_{r=0}^{2} C_r^2 a^r b^{2-r} = C_0^2 a^0 b^{2-0} + C_1^2 a^1 b^{2-1} + C_2^2 a^2 b^{2-2}$$

$$= b^2 + 2ab + a^2$$

EX(2) let $n=3$, $n=4$; find $(a+b)^n$ H.w.

EX(3) let $a=1$, $b=1$, find $(a+b)^n$ or prove that

$$\sum_{i=0}^{n} C_i^n = 2^n$$

Sol $\because (a+b)^n = \sum_{i=0}^{n} C_i^n a^i b^{n-i}$, $\therefore (1+1)^n = \sum_{i=0}^{n} C_i^n 1^i 1^{n-i}$

$$\therefore 2^n = \sum_{i=0}^{n} C_i^n$$

EX: prove that $3^n = \sum_{i=0}^{n} C_i^n 2^i$ H.w.