

2. Combination

It's a selection of different objects (r), taken from a set of different objects (n) without required to the arrangement of them it's given by:-

$$C_r^n = \frac{n!}{r!(n-r)!} = \frac{P_r^n}{r!}, \quad r \leq n.$$

Ex: How many ways can be selected (chosen).
Committee of four persons from six?

Sol $n = 6, r = 4;$

$$\therefore C_r^n = C_4^6 = \frac{6!}{4! (6-4)!} = \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2!} = 15$$

= 15 no. of selected

مسعودی

Ex(2) we want to select committee of six members from five teachers and ten students; find

- ① How many ways can be selected?
- ② " " " " " s.t. no. of teachers are always more than the students?
- ③ There are at most 2 students?

امثال (9)

Sol ① $n = 5 + 10 = 15$, $r = 6$,

$${}^n C_r = {}^{15} C_6 = \frac{15!}{6!(15-6)!} = 5005 \text{ no. of selec.}$$

$$\begin{aligned} \text{② no. of selec} &= {}^5 C_5 {}^{10} C_1 + {}^5 C_4 {}^{10} C_2 \\ &= 10 + 225 \\ &= 235 \end{aligned}$$

$$\text{③ no. of selec.} = {}^{10} C_1 {}^5 C_5 + {}^{10} C_2 {}^5 C_4 = 235$$

if he want ④ There are at least 2 students:-

$$\begin{aligned} \text{no. of selec.} &= {}^{10} C_2 {}^5 C_4 + {}^{10} C_3 {}^5 C_3 + {}^{10} C_4 {}^5 C_2 + {}^{10} C_5 {}^5 C_1 \\ &= \boxed{4785} \text{ H.W.} \end{aligned}$$

⑤ no. of students are always more than the teachers?

$$\begin{aligned} \text{no. of selec.} &= {}^{10} C_5 {}^5 C_1 + {}^{10} C_4 {}^5 C_2 = \boxed{3360} \text{ H.W.} \end{aligned}$$

Ex: Find the value of n if :-

$$\begin{aligned} \textcircled{1} \quad {}^nC_2 = 3 &\Rightarrow \frac{(n+1)!}{2! (n+1-2)!} = 3 \\ &\Rightarrow \frac{(n+1)(n)(\cancel{n-1})!}{2! (n-1)!} = 3 \end{aligned}$$

$$\therefore n^2 + n - 6 = 0$$

$$\Rightarrow (n+3)(n-2) = 0$$

$$\therefore n+3 = 0 \Rightarrow n = -3 \text{ neglect}$$

$$\text{or } n-2 = 0 \Rightarrow \boxed{n = 2}$$

notes: $\textcircled{1} \quad {}^nC_n = 1$

$$\therefore {}^nC_n = \frac{\cancel{n!}}{\cancel{n!} (n-n)!} = \frac{1}{0!} = 1$$

$$\textcircled{2} \quad {}^nC_1 = n \Rightarrow \therefore {}^nC_1 = \frac{n!}{1! (n-1)!} = \frac{n(\cancel{n-1})!}{(n-1)!} = n$$

$$\textcircled{3} \quad {}^nC_0 = 1 \Rightarrow \therefore {}^nC_0 = \frac{n!}{0! (n-0)!} = 1$$