Total probability theorem

احتمالية / ص (27)

متتالية من الأحداث المتنافية والشاملة

Suppose that $A_1, A_2, \dots, A_n$ are ~~mutual~~ mutually exclusive (m.e.) events;

s.t.: $A_1 \cup A_2 \cup \dots \cup A_n = S$ and $P(A_i) > 0, \forall i = 1, 2, \dots, n$

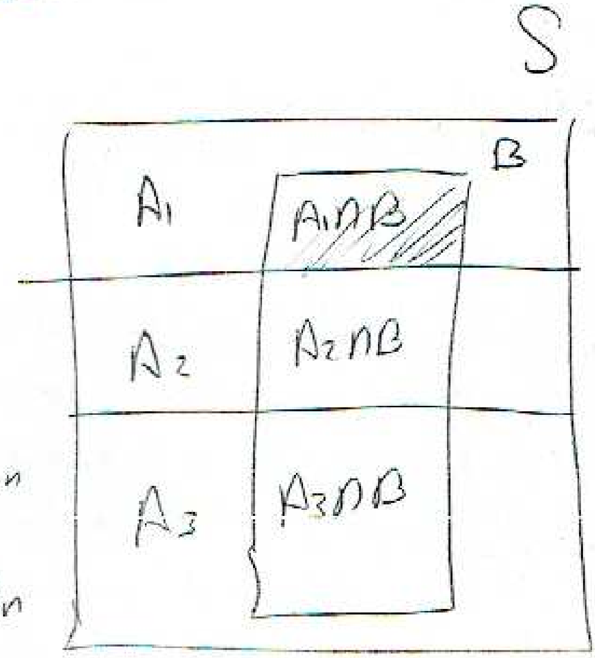
Then for any event B; s.t.

$$B = B \cap S \text{ we have:-}$$

$$P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + \dots + P(A_n) \cdot P(B/A_n)$$

$$\therefore \boxed{P(B) = \sum_{i=1}^{n} P(A_i) \cdot P(B/A_i)} \to \text{Total prob.}$$

notes:



1- If: $A_1, A_2, \dots, A_n$ are mutually exclusive (M.e) events, then:-

a- $A_i \cap A_j = \phi, \forall i \neq j, i, j = 1, 2, \dots, n$

b- $\bigcup_{i=1}^{n} A_i = S = A_1 \cup A_2 \cup A_3 \dots \cup A_n$

2) $\because B = B \cap S = BS = B(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n)$

$$= BA_1 \cup BA_2 \cup BA_3 \cup \dots \cup BA_n$$

$$\therefore P(B) = P(BA_1) + P(BA_2) + P(BA_3) + \dots + P(BA_n)$$

$$\therefore P(B) = P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + \dots + P(A_n) \cdot P(B/A_n).$$

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$$\therefore P(B|A_1) = \frac{P(B \cap A_1)}{P(A_1)}$$

$$\boxed{\therefore P(B) = \sum_{i=1}^{n} P(A_i)\, P(B|A_i)} \rightarrow \text{Total prob.}$$

مثال

Ex: (1) Two urns, Urn I Contains 5 white and 7 red balls.
Urn II Contains 6 white and 4 red balls, one of the urns is selected at <u>random</u> and a ball is <u>drawn from</u> it. Find the prob. that the ball drawn will be white.

Sol A_1 = Urn I is chosen

A_2 = " II " "

B = white ball is drawn

	w	r
I	5	7
II	6	4

$n(S) = C_1^{12}$, $n(B) = C_1^{5}$; $n(S) = C_1^{10}$, $n(B) = C_1^{6}$

$B = (B \cap A_1) \cup (B \cap A_2)$

$\therefore P(A_1) = P(A_2) = \frac{1}{2}$

$\therefore P(B) = P(BA_1) + P(BA_2)$

$= P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2)$

$\therefore P(B|A_1) = \frac{C_1^5}{C_1^{12}} = \frac{5}{12}$; $P(B|A_2) = \frac{C_1^6}{C_1^{10}} = \frac{6}{10}$

$\therefore P(B) = \frac{1}{2} \cdot \frac{5}{12} + \frac{1}{2} \cdot \frac{6}{10} = 0.51 \rightarrow$ Total prob.