H.W? If $p(x)$ is p.m.f. of the r.v. X, Find the Constant C for :-

سؤال

$$p(x) = \begin{cases} C\left(\frac{1}{2}\right)^x, & x = 1,2,3,\dots \\ 0, & o.w \end{cases}$$

Sol

$$\sum_{\forall x} p(x) = 1$$

$$\sum_{x=1}^{\infty} C\left(\frac{1}{2}\right)^x = 1$$

$$C\sum_{x=1}^{\infty}\left(\frac{1}{2}\right)^x = 1$$

$$C \cdot \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

$$\frac{\frac{C}{2}}{\frac{1}{2}} * \frac{1}{1}$$

$$\frac{C}{2} = \frac{1}{2}$$

$$2C = 2$$

$$\boxed{C = 1}$$

$$\sum_{x=0}^{\infty} q^x = \frac{1}{1-q}$$

$$1 + \sum_{x=1}^{\infty} q^x = \frac{1}{1-q}$$

$$\sum_{x=1}^{\infty} q^x = \frac{1}{1-q} - 1$$

$$= \frac{1-(1-q)}{1-q}$$

$$= \frac{q}{1-q}$$

The Cumulative Distribution Function (C.D.F.) الدالة التراكمية (40)

The Cumulative prob. function of r.v. X is given by

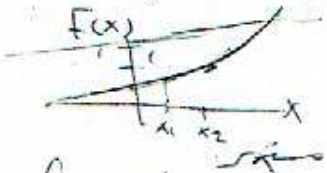
$F(x) = P_r(X \leq x)$ for any real number x

$$= \begin{cases} \sum_{-\infty}^{x} p(k) \longrightarrow \text{D.r.v.} \\ \int_{-\infty}^{x} f(k)\,dk \longrightarrow \text{C.r.v.} \end{cases}$$

properties of (C.D.F.). $F(x)$

① $\lim_{x \to \infty} F(x) = F(\infty) = P_r(X \leq \infty) = 1$

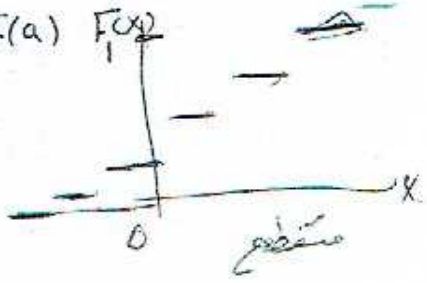
2) $\lim_{x \to -\infty} F(x) = F(-\infty) = P_r(X \leq -\infty) = 0$



3) If $x_1 < x_2$, then $F(x_1) < F(x_2)$

4) $P(a < x < b) = F(b) - F(a)$, ($a, b$ are real numbers.)

(5) $P(X > a) = P(a < x < \infty) = F(\infty) - F(a) = 1 - F(a)$



Ex 6) Find C.D.F. of p.m.f

$$P(x) = \begin{cases} \frac{1}{8} & ; x = 0 \\ \frac{3}{8} & ; x = 1 \\ \frac{3}{8} & ; x = 2 \\ \frac{1}{8} & ; x = 3 \\ 0 & ; o.w. \end{cases}$$