

Ex(2) Find $V(X)$ or σ^2

for the r.v. X where

$$f(x) = \begin{cases} 3(1-x)^2 & , 0 < x < 1 \\ 0 & , o.w. \end{cases}$$

properties of Variance

1- $V(c) = 0$, c is constant

2- $V(cX) = c^2 V(X)$, c is constant

3- $V(X \mp c) = V(X)$

4- $V(aX \mp c) = a^2 V(X)$

5- $\sigma_X = \sqrt{V(X)} \rightarrow$ standard deviation of X
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H.w.

Q) Find ① $V(X-3) = V(X)$ خاصية رقم (3)

② $\text{Var}(\frac{3}{2}X) = (\frac{3}{2})^2 V(X)$ خاصية رقم (2)

3) $\text{Var}(-14X+1) = (-14)^2 V(X)$ خاصية رقم (4)

المثال رقم (2)

احتمالية رقم (5)

$$E(X^2) = \int_0^1 x^2 \cdot 3(1-x)^2 dx$$

$$= \int_0^1 x^2 \cdot 3(1-2x+x^2) dx$$

$$= \int_0^1 3x^2 - 6x^3 + 3x^4 dx$$

$$H.w. = \left[\frac{3x^3}{3} - 6 \frac{x^4}{4} + 3 \frac{x^5}{5} \right]_0^1$$

$$= \frac{4}{4} - \frac{6}{4} + \frac{3}{5}$$

$$= \frac{-2}{4} + \frac{3}{5} = \frac{-10+12}{20} = \frac{2}{20}$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= \frac{2}{20} - \left(\frac{1}{4}\right)^2$$

$$= \frac{2}{20} - \frac{1}{16}$$

Moments

المoment (52)

If X Random Variable, then the moment of the degree r of the Variable X around (a) is

$$E(X-a)^r$$

such that

r : positive integer number

a : Constant

There are two important Cases:-

① Moment r of the Random Variable X around the origin point (Zero), where $a=0$ denoted by M_r
Such that

$$M_r = E(X-a)^r = E(X^r)$$

$$\text{i.e. } r=0 \Rightarrow E(X^0) = M_0 = 1$$

$$r=1 \Rightarrow E(X) = M_1$$

$$r=2 \Rightarrow E(X^2) = M_2$$

$$r=3 \Rightarrow E(X^3) = M_3$$

② if $a = \mu$, then the central moment of the degree r around the mean (μ) for the Random Variable X is M_r

$$M_r = E(X-\mu)^r$$

$$M_1 = E(X-\mu)$$

$$= E(X) - \mu$$

$$= \mu - \mu = 0$$