$$\mu_2 = E(X-\mu)^2$$
$$= E(X^2 - 2X\mu + \mu^2)$$
$$= E(X^2) - 2\mu E(X) + \mu^2$$
$$= E(X^2) - 2\mu^2 + \mu^2$$
$$= E(X^2) - \mu^2$$
$$= E(X^2) - [E(X)]^2$$
$$= Var(X)$$
$$= m_2 - m_1^2$$

$$\mu_3 = E(X-\mu)^3 \quad \underline{\underline{H.w}}$$

i.e

$k=0$: $E(X)^{k=0} = 1$, $E(X-\mu)^{k=0} = 1$

$k=1$: $E(X)^{k=1} = \mu = E(X)$, $E(X-\mu)^{k=1} = \text{zero}$

$\rightarrow = E(X) - \mu$
$= E(X) - E(X)$
$= 0$

$k=2$: $E(X^2) = Var(X) + \{E(X)\}^2$

$E(X-\mu)^2 = E(X^2) - 2\mu E(X) + \mu^2$
$= E(X^2) - 2\mu^2 + \mu^2$
$= E(X^2) - \mu^2$
$= E(X^2) - \{E(X)\}^2$
$= Var(X)$

<u>Note:</u> There are some rules

① $\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$

② $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

3) $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Ex: If X random variable with p.m.f. defined as :-

$$p(x) = \frac{1}{n} \quad , x = 1, \dots, n$$

Find m_r , μ_r when $r = 0, 1, 2$

sol

$$m_r = E(X^r)$$
$$m_0 = E(X^0) = 1$$

$$m_1 = E(X)$$

$r = 1$

$$m_1 = E(X) = \sum_{x=1}^{n} x\,p(x)$$
$$= \sum_{x=1}^{n} x \cdot \frac{1}{n}$$
$$= \frac{1}{n} \sum_{x=1}^{n} x$$
$$= \frac{1}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{2}$$
$$= \frac{n+1}{2}$$

$r = 2$

$$m_2 = E(X^2) = \sum_{x=1}^{n} x^2 p(x)$$
$$= \sum_{x=1}^{n} x^2 \cdot \frac{1}{n} = \frac{1}{n} \sum_{x=1}^{n} x^2$$
$$= \frac{1}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6}$$
$$= \frac{(n+1)(2n+1)}{6}$$

$$\mu_r = E(X - m)^r$$
$$\mu_0 = E(X - \mu)^0 = 1$$

$$\mu_1 = E(X - \mu)^1$$
$$= E(X) - \mu$$
$$= \mu - \mu = 0$$

$$\mu_2 = E(X - \mu)^2$$
$$= E(X^2) - (E(X))^2$$
$$= m_2 - m_1^2$$
$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$
$$= \frac{4(n+1)(2n+1) - 6(n+1)^2}{24}$$
$$= \frac{2(n+1)[2(2n+1) - 3(n+1)]}{24}$$
$$= \frac{(n+1)}{12}(4n+2-3n-3)$$
$$= \frac{(n+1)(n-1)}{12}$$
$$= \frac{n^2-1}{12}$$