سؤال

الامتحانات/قسم (55)

Q1). Find the mean and Variance of r.v. X whose density is

$$f(x) = \begin{cases} 2x , & 0 < x < 1 \\ 0 , & o.w. \end{cases}$$

Sol

$$E(X) = m_1 = \int_{\forall x} x \cdot f(x)\,dx = \int_0^1 x \cdot 2x\,dx = 2\frac{x^3}{3}\Big]_0^1 = \frac{2}{3}$$

$$Var(X) = \sigma_X^2 = E(X-\mu)^2 = E(X-E(x))^2$$

$$= E(x^2) - \{E(x)\}^2$$

$$= m_2 - m_1^2$$

$$E(x^2) = \int_{\forall x} x^2 \cdot f(x)\,dx = \int_0^1 x^2 \cdot (2x)\,dx = \frac{2}{4}x^4\Big]_0^1 = \frac{2}{4} = \frac{1}{2}$$

$$\sigma_X^2 = E(x^2) - \{E(x)\}^2$$

$$= \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{18}$$

Q2) let the density of X be given by:-

X	-1	0	1	2
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{4}$

Find the mean and Variance of X.

Sol

$$\mu = m_1 = E(X) = \sum_{\forall x} x\,p(x) = -1 \cdot \frac{1}{8} + 0 \cdot \frac{3}{8} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4}$$

$$= \frac{7}{8}$$

$$E(x^2) = \sum_{\forall x} x^2 p(x) = (-1)^2 \cdot \frac{1}{8} + (0)^2 \cdot \frac{3}{8} + (1)^2 \cdot \frac{1}{4} + (2)^2 \cdot \frac{1}{4}$$

$$= \frac{11}{8}$$

$$\frac{1}{8} + \frac{1}{4} + 1$$

Moment generating function (m.g.f.)

The moment generating function for the r.v. X is denoted by $M_x(t)$ such that

$$M_x(t) = E(e^{tx})$$

$$= \begin{cases} \sum_{\forall x} e^{tx} p(x) & \text{D.r.v.} \\ \int_{\forall x} e^{tx} f(x)\,dx & \text{C.r.v.} \end{cases}$$

Ex: Find $M_x(t)$ for the r.v. X ~~[illegible]~~.

IF $p(x) = \frac{1}{6}$, $x = 1, 2, 3, 4, 5, 6$

sol

$$M_x(t) = E(e^{tx}) = \sum_{x=1}^{6} e^{tx} p(x)$$

$$= \sum_{x=1}^{6} e^{tx} \cdot \frac{1}{6}$$

$$= \frac{1}{6} \sum_{x=1}^{6} e^{tx}$$

$$= \frac{1}{6} \left[e^{t} + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t} \right]$$

$$M_x(t) = \frac{e^{t}}{6} \left[1 + e^{t} + e^{2t} + e^{3t} + e^{4t} + e^{5t} \right]$$

Example let the p.d.f. of the r.v. X is defined as :-

$$f(x) = \frac{1}{6} \quad , \quad 0 < x < 6$$

Find $M_x(t)$?

sol

$$M_x(t) = E(e^{xt})$$

$$= \int_0^6 e^{tx} \cdot \frac{1}{6}\, dx$$

$$= \frac{1}{6} \cdot \frac{1}{t} \int_0^6 t\, e^{tx}\, dx$$

$$= \frac{1}{6t}\, e^{tx} \Big]_0^6 = \frac{1}{6t} e^{6t} - \frac{1}{6t}$$

$$M_x(t) = \frac{1}{6t}\left[e^{6t} - 1\right] = m.g.f$$

Theorem: The r^{th} moment can be obtained from $M_x(t)$ by :-

$$\left.\frac{d^r M_x(t)}{dt^r}\right|_{t=0} \quad , \quad r = 1, 2, \ldots, n$$

i.e

$$\left.\frac{d M_x(t)}{dt}\right|_{t=0} = m_1 = E(x)$$