$$\frac{d^2 M_x(t)}{dt^2}\bigg|_{t=0} = m_2 = E(x^2)$$

$\vdots$

$\vdots$

$$e^{x} = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{tx} = \sum_{k=0}^{\infty} \frac{(tx)^k}{k!}$$

$$M_x(t) = E(e^{tx}) = E\left[\sum_{k=0}^{\infty} \frac{(tx)^k}{k!}\right]$$

$$= E\left[1 + tx + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + \cdots\right]$$

$$= 1 + t\,E(x) + t^2 \frac{E(x^2)}{2!} + t^3 \frac{E(x^3)}{3!} + \cdots$$

$$= 1 + t\,m_1 + t^2 \cdot \frac{m_2^*}{2!} + t^3 \frac{m_3}{3!} + \cdots$$

$$M'_x(t) = m_1 + t\,m_2 + 3t^2 \cdot \frac{m^3}{3!} + 4t^3 \frac{m_4}{4!} + \cdots$$

$$M'_x(t)\bigg|_{t=0} = m_1 + 0 + 0 + \cdots$$

$$M''_x(t) = 0 + m_2 + 6t \cdot \frac{m_3}{3!} + \frac{12}{4!} t^2 m_4 + \cdots$$

$$M''_x(t)\bigg|_{t=0} = 0 + m_2 + 0 + 0 + \cdots$$

$\vdots$

$$\frac{d^r M_x(t)}{dt^r}\bigg|_{t=0} = m_r^*.$$

احتمالية رقم (59)

Example let the D.r.v. X has the following p.m.f.

$$P(x) = \left[\frac{1}{2}\right]^x , \quad x = 1, 2, 3, \ldots$$

باستخدام الدالة المولدة للعزوم

Find 1. $E(x)$, 2. $Var(x)$

Sol

$$M_x(t) = E\left(e^{tx}\right)$$

$$= \sum_{x=1}^{\infty} e^{tx} \cdot \left[\frac{1}{2}\right]^x$$

$$= \sum_{x=1}^{\infty} \left[\frac{e^{t}}{2}\right]^x$$

$$= \frac{e^t}{2} + \left[\frac{e^t}{2}\right]^2 + \left[\frac{e^t}{3}\right]^3 + \ldots$$

متسلسلة هندسية

$$* \sum_{x=1}^{\infty} q^x = \frac{q}{1-q}$$

$$M_x(t) = \frac{e^t/2}{1 - e^t/2} = \frac{e^t}{2 - e^t}$$

$$\frac{(2-1)\cdot 1 + 1\cdot 1}{(2-1)^2}$$

$$\frac{2}{1}$$

$$\frac{2e^0}{(2-e^0)^2}$$

$$\frac{d}{dt} M_x(t) \Big|_{t=0} = \frac{(2-e^t)\cdot e^t - e^t\cdot(-e^t)}{(2-e^t)^2}\Bigg|_{t=0}$$

$$= m_1 = E(x) = \mu$$

$$= \frac{e^t\,(2 \cancel{- e^t} \cancel{+ e^t})}{(2-e^t)^2}\Bigg|_{t=0} = \frac{2e^t}{(2-e^t)^2}\Bigg|_{t=0} = \frac{2}{1}$$