# 1. Physics Laws for The Neutral Atmosphere

Characterizing the atmosphere by the radio waves are propagated leads to a subdivision troposphere and ionosphere. The ionosphere, the upper part of the atmosphere, is the dispersive medium (the propagation delay is frequency dependent) whereas the troposphere is non-dispersive. The troposphere is also referred to as neutral atmosphere.

In this chapter, the following physical laws and equations are given the gas equation (equation of state), hydrostatic equation, and Snell’s law

* 1. **Equation of state**

Laboratory experiments show that the pressure, volume, and temperature of any material can be related by an equation of state over a wide range of conditions.

All gases are found to follow approximately the same equation of state, which is referred to as the ideal gas equation. For most purposes, we may assume that atmospheric gases, whether considered individually or as a mixture, obey the ideal gas equation exactly. This section considers various forms of the ideal gas equation and its application to dry and moist air.

There are three laws for gases are given:

* Charles’ constant pressure law: “At constant pressure for a rise in temperature of 1degree Celsius, all gases expand by a constant amount, equal to 1/ 273 of their volume at 0 degree Celsius”.
* Charles’ constant volume law: “If the volume is kept constant, all gases undergo an increase in pressure equal to 1/ 273 of their pressure at 0 degree Celsius”.
* Boyle’s law: “At constant temperature the product of pressure and volume is constant”.

Based on these laws, the gas equation of state is formulated for perfect gases:

$PV=mRT$ (1)

where P, V, m, and T are the pressure (Pa), volume (m3), mass (kg), and absolute temperature (in kelvin, K, where K = °C + 273.15) of the gas, respectively, and R is a constant (called the gas constant) for 1 kg of a gas. The value of R depends on the particular gas under consideration. Because $m/V=ρ$, where $ρ$ is the density of the gas, the ideal gas equation may also be written in the form

$P=ρRT$ (2)

For a unit mass (1 kg) of gas m=1 and we may write (2) as:

$Pα=RT$ (3)

where $α=1/ρ$ is the *specific volume* of the gas, i.e., the volume occupied by 1 kg of the gas at pressure p and temperature T.

If the temperature is constant (1) reduces to Boyle’s law, which states if the temperature of a fixed mass of gas is held constant, the volume of the gas is inversely proportional to its pressure. Changes in the physical state of a body that occur at constant temperature are termed isothermal. Also, implicit in (1) are Charles’ two laws. The first of these laws states for a fixed mass of gas at constant pressure, the volume of the gas is directly proportional to its absolute temperature. The second of Charles’ laws states for a fixed mass of gas held within a fixed volume, the pressure of the gas is proportional to its absolute temperature.

We define now a gram-molecular weight or a mole (abbreviated to mol) of any substance as the molecular weight, M, of the substance expressed in grams. For example, the molecular weight of water is 18.015; therefore, 1 mol of water is 18.015 g of water. The number of moles *n* in mass *m* (in grams) of a substance is given by

$$n=\frac{m}{M} (4)$$

Because the masses contained in 1 mol of different substances bear the same ratios to each other as the molecular weights of the substances, 1 mol of any substance must contain the same number of molecules as 1 mol of any other substance. Therefore, the number of molecules in 1 mol of any substance is a universal constant, called Avogadro’s number, NA. The value of NA is $6.022×10^{23}$ per mole.

According to *Avogadro’s hypothesis*, gases containing the same number of molecules occupy the same volumes at the same temperature and pressure. It follows from this hypothesis that provided we take the same number of molecules of any gas, the constant R in (1) will be the same. However, 1 mol of any gas contains the same number of molecules as 1 mol of any other gas. Therefore, the constant R in (1) for 1 mol is the same for all gases; it is called the *universal gas constant* (R\*). The magnitude of R\* is$ 8.3145 J K^{-1} mol^{-1}$. The ideal gas equation for 1 mol of any gas can be written as

$PV=R^{\*}T$ (5)

and for *n* moles of any gas as

$PV=nR^{\*}T$ (6)

The gas constant for one molecule of any gas is also a universal constant, known as Boltzmann’s constant, k.

Because the gas constant for NA molecules is R\*, we have

$$k=\frac{R^{\*}}{N\_{A}} (7)$$

Hence, for a gas containing *n0* (number density) molecules per unit volume, the ideal gas equation is

Prove??

 $P=n\_{0}kT$ (8)

If the pressure and specific volume of dry air (i.e., the mixture of gases in air, excluding water vapor) are $P\_{d} $and$ α\_{d}$, respectively, the ideal gas equation in the form of (3) becomes

 $P\_{d}α\_{d}=R\_{d}T$ (9)

where *Rd* is the gas constant for 1 kg of dry air. By analogy with (4), we can define the *apparent molecular weight Md* of dry air as the total mass (in grams) of the constituent gases in dry air divided by the total number of moles of the constituent gases; that is,

$$M\_{d}=\frac{\sum\_{i}^{}m\_{i}}{\sum\_{i}^{}\frac{m\_{i}}{M\_{i}}} (10)$$

where *mi* and *Mi* represent the mass (in grams) and molecular weight, respectively, of the ith constituent in the mixture. The apparent molecular weight of dry air is 28.97. Because *R\** is the gas constant for 1 mol of any substance, or for *Md* (= 28.97) grams of dry air, the gas constant for 1 g of dry air is $ R^{\*}/M\_{d }$, and for 1 kg of dry air it is

$$R\_{d}=1000\frac{R^{\*}}{M\_{d}}=1000\frac{8.3145}{28.97}=287.0 JK^{-1}Kg^{-1} \left(11\right)$$

The ideal gas equation may be applied to the individual gaseous components of air. For example, for water vapor (3) becomes

$$eα\_{v}=R\_{v}T (12)$$

where *e* and $α\_{v}$ are, respectively, the pressure and specific volume of water vapor and $R\_{v}$ is the gas constant for 1 kg of water vapor. Because the molecular weight of water is $M\_{w}(=18.016)$ and the gas constant for $M\_{w}$ grams of water vapor is *R\**, we have

$$R\_{v}=1000\frac{R^{\*}}{M\_{w}}=1000\frac{8.3145}{18.016}=461.51 JK^{-1}Kg^{-1} \left(13\right)$$

From (11) and (13),

$$\frac{R\_{d}}{R\_{v}}=\frac{M\_{w}}{M\_{d}}=ε=0.622 (14)$$

Because air is a mixture of gases, it obeys Dalton’s law of partial pressures, which states the total pressure exerted by a mixture of gases that do not interact chemically is equal to the sum of the partial pressures of the gases. The partial pressure of a gas is the pressure it would exert at the same temperature as the mixture if it alone occupied all of the volume that the mixture occupies.

**Exercise 1**: If at 0 °C the density of dry air alone is $1.275 kg m^{-3} $and the density of water vapor alone is $4.770 × 10^{-3}kg m^{-3} $, what is the total pressure exerted by a mixture of the dry air and water vapor at 0 °C?

**Solution:** From Dalton’s law of partial pressures, the total pressure exerted by the mixture of dry air and water vapor is equal to the sum of their partial pressures. The partial pressure exerted by the dry air is, from (9),

$$P\_{d}=\frac{1}{α\_{d}}R\_{d}T=ρ\_{d}R\_{d}T$$

where $ρ\_{d}$ is the density of the dry air ($1.275 kg m^{-3} at 273 K$), $R\_{d}$ is the gas constant for 1 kg of dry air ($287.0 J K^{-1} kg^{-1}$), and *T* is 273.2 K. Therefore,

$$P\_{d}=9.997×10^{4}Pa=999.7 hPa$$

Similarly, the partial pressure exerted by the water vapor is, from (12),

$$e=\frac{1}{α\_{v}}R\_{v}T=ρ\_{v}R\_{v}T$$

where $ρ\_{v}$ is the density of the water vapor ($4.770 ×10^{-3} kg m^{-3} at 273 K$), $R\_{v}$ is the gas constant for 1 kg of water vapor ($461.5 J K^{-1} kg^{-1}$), and *T* is 273.2 K. Therefore,

$$e=601.4Pa=6.014 hPa$$

Hence, the total pressure exerted by the mixture of dry air and water vapor is (999.7+6.014) hPa or 1006 hPa.

**Exercise 2**: Determine the apparent molecular weight of the Venusian atmosphere, assuming that it consists of 95% of CO2 and 5% N2 by volume. What is the gas constant for 1 kg of such an atmosphere? (Atomic weights of C, O, and N are 12, 16, and 14, respectively.)

**Solve??**