1. **More Results on Riemann Integral**

(8.1) **Example**: Let represents a set of rational numbers in . define a function as . We note .

Since is finite set

is neglected set

is Riemann integral on

Also, .

(8.2) **Example**: Let a function defined as . We note , but does not Riemann integral on and this convergent not uniformly.

(8.3) **Example**: define a function as , continuous and then is Riemann integral on , also

converges to , also

this function is Riemann integral on , and

, but , also

(8.4) **Theorem**: If sequence of Riemann integral bounded functions on and , then is Riemann integral on , and , this means .

**Proof:** since and bounded on

bounded on

Since is Riemann integral on

is neglected set

Put is neglected set

continuous on

Since continuous on this means

is neglected set

is Riemann integral on

**Differentiation and Riemann Integration**

(8.5) **Theorem**: Let open interval in and let Riemann integral bounded function on all closed sub interval of . If , then defined as continuous.

**Proof:** since bounded

Put sup

, we get

If , take

If

uniformly continuous.

(8.6) **Theorem**: Let open interval in and let continuous. If , then defined as differentiable and

**Proof:** let **,** and let

By using mean value theorem, we have

Since , if , then

(8.7) **Theorem**: Let continuous function and let function , then , we get .

**Proof:** define a function as

differentiable on , and

Since

(8.8) **Corollary:** Let continuous function, then there is differentiable such that .