1. **Riemann and Lebesgue**
   1. **Theorem**: Let is a bounded function, then Riemann integral on a partition on .

**Proof:** let is Riemann integral on .

Let , since inf sup

By using definition of inf partition of .

By using definition of sup partition of .

Put , since

Since

.

let a partition on .

Since of , we get

and

Since

* 1. **Theorem**: Every continuous function defined on closed interval is Riemann integral.
  2. **Example**: let a function defined as

, show that Riemann integral on .

**Solution: ,** let

Let partition of

Let partition of

Put

inf

If

Since , but this is contradiction with Archimedes property

So, we have

Therefore, Riemann integral on .

**Lebesgue Theorem in Riemann Integral**

* 1. **Definition:** Let . We said that a negligible if countable family of open intervals

1. (2)
   1. **Theorem**:
2. Every countable set of real numbers is negligible.
3. Every subset of negligible set is negligible.
4. A union of any countable family of negligible sets is negligible set.

**Proof:** (1) let countable set .

1. If a finite

Let

Take

is open interval contains on

1. If an infinite

Let take

is open interval contains on

* 1. **Corollary:**

1. Every interval in does not negligible.
2. Every subset in contains on interval as subset is not negligible.
3. in is negligible, since is countable.
4. is not negligible.
5. in is not negligible.
   1. **Definition:** Let interval and bounded function for all open interval in , put

sup

Put, , then . Saltus of a function at denoted by and defined as

* 1. **Theorem**: A function is a continuous at .

**Proof:** let .

Let by definition of inf, there is open interval

is continuous at .

let is continuous at .

Let an open interval