**15. Continuity and Compactness**

(15.1)**Theorem**: Let are metric spaces and is a continuous function. If is a compact set , then is a compact set in .

**Proof:** let is an open cover of in .

Since

Since is a continuous is an open set in

is an open cover of

Since is a compact space

is a compact set in .

(15.2)**Corollary**: Let are metric spaces and is a continuous function. If is a compact set in , then is a compact set in .

(15.3)**Example:** Let is usual metric space and is defined as .

We note that is a continuous, since is a constant and is a compact in , since is a finite, but does not compact.

(15.4)**Theorem**: Let are metric spaces , then is a compact space is a compact space.

**Proof:** since .

Let is a compact space, since is a continuous is a compact in .

Since is a bijective is a compact space.

Now, let is a compact space

Since is a continuous is a compact.

(15.5)**Theorem**: Let is a compact space and is a continuous function, then

1. is a bounded.
2. If inf sup , then .

**Proof:** (1) since is a compact space and is a continuous is a compact set in .

Since every compact set in is a closed and bounded is a bounded.

(2) since is a bounded and since is a closed

Put .

(15.6)**Theorem**: Let are a metric spaces and is a continuous function. If is a compact space, then is an uniform continuous.

(15.7)**Corollary**: Let is usual metric space. If is a continuous function, then is an uniform continuous.