**14. Compactness**

(14.1)**Definition**: Let is a family of subsets in , and let . We said that is a covering of , if . If is a finite, then is a finite covering of .

(14.2)**Example:** Let , then

1. A family represents a covering of , since .
2. A family does not represent a covering of , since .
3. A family represents a covering of and .

(14.3)**Example:**

1. A family represents an infinite covering of .
2. A family represents an infinite covering of .
3. A family does not represent a covering of .

(14.4)**Definition**: Let are covering of , we said that is a sub covering from , if for all .

(14.5)**Example:** Each of are covering of and is a subfamily of .

(14.6)**Definition**: Let is a subset of and let is a covering of . We said that is an open cover, if is an open set in .

(14.7)**Example**: In . Prove that a family is an open cover of .

**Solution:** let ,

Since (by Archimedes property).

Since

is a covering of .

Since is open set is an open set of .

(14.8)**Example**: In , we have

, , are an open cover of , also is a sub cover of.

(14.9)**Example**: Let be discrete metric space and . Prove that is an open cover of .

**Solution:** since is a covering of .

Since is discrete metric space an open set in

is an open cover of .

(14.10)**Definition**: Let is metric space and let . We said that is a compact set in , if for all open cover contains a finite sub covering.

(14.11)**Example**: In , we have

1. does not compact in .
2. is a compact in .
3. A space does not compact.

**Solution:** (1) Take is an open cover of , but does not contain on a finite sub cover does not compact.

(14.12) **Example**: Every indiscrete metric space is a compact, since an unique open cover of is .

(14.13) **Theorem:** Every finite set in a metric space is a compact.

**Proof:** let is a finite set in

Let is a open cover of in .

is an open set in .

Since

is a finite sub covering from of .

is a compact set.

(14.14)**Example**: Let is discrete metric space, then is a compact is a finite.

(14.15)**Theorem**: Let is a subspace of a metric space and , then is a compact in is a compact in .

(14.16)**Theorem**: Every closed set in a compact metric space is a compact.

(14.17)**Theorem**: Every compact set in a metric space is a closed and bounded.

(14.18)**Definition:** We said that a family of sets that satisfy a finite intersection property, if intersection every finite subfamily is a non- empty set.

(14.19)**Theorem;** A metric space is a compact if every family of a closed sets satisfies a finite intersection property, then its non-empty set.

(14.20)**Definition:** We said that a metric space is a countable compact, if for all open cover and countable in contains on a finite sub covering.

(14.21)**Theorem;** A metric space is a countable compact every countable family of a closed sets and satisfy a finite intersection property is a non- empty intersection.