**9. Interior Points**

(9.1) **Definition**: Let be a metric space, and . We say that a point is an interior point in , if an open set in . Or if .

(9.2) **Definition**: The set of all interior points in is denoted by

, so .

(9.3) **Notes**: From previous definitions, we deduce

1. is an open set in .
2. is an open set in iff .
3. .

(9.4) **Example:** Let be an indiscrete metric space, and . Calculate .

**Solution:** since be an indiscrete metric space .

(9.5) **Example:** Let be a discrete metric space, and . Calculate .

**Solution:** since be a discrete metric space is an open set in .

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(9.6) **Example:** Let be usual metric space, and , then we have

1. If .
2. If .
3. If .
4. If .
5. If .
6. If is a finite, then .
7. If .
8. If .
9. If .
10. If .

(9.7) **Theorem:** Let be a metric space, and , then we have

1. If .
2. .
3. .

**Proof:** (1) let .

Since .

(9.8) **Note:** Not necessary that , for example

Let be usual metric space, and let , we have

, but

and .

(9.9) **Definition:** Let be a metric space, and . We said that is a closure point of set , if .

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(9.10) **Notes:** From previous definitions, we deduce

1. is a closed set in .
2. is a closed set .
3. .

(9.11) **Example:** Let be indiscrete metric space, and . Calculate .

**Solution:** since be indiscrete metric space

(9.12) **Example:** Let be discrete metric space, and . Calculate .

**Solution:** since be discrete metric space is a closed set in .

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(9.13) **Example:** Let be usual metric space, and , then we have

1. If .
2. If .
3. If .
4. If .
5. If .
6. If .
7. If .
8. If .
9. If .

(9.14) **Theorem:** Let be a metric space, and , then we have

1. If .
2. .
3. .

**Proof:** (1) let .

Since .

(9.15) **Note:** Not necessary that , for example

Let be usual metric space, and let , we have

, but

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(9.16) **Definition:** Let be a metric space, and . We said that is a limit point of set , if . The set of all limit points denoted by .

(9.17) **Definition:** We said that is an isolated point of , if .

(9.18) **Definition:** We said that the set is an isolated set, if .

(9.19) **Definition:** We said that the set is a perfect set, if .

(9.20) **Definition:** We said that the set is a dense set, if .

(9.21) **Theorem:** Let be a metric space, and , then we have

1. If .
2. If .
3. .
4. .
5. If is a closed in .
6. is a closed in .

**Proof:** (1) Since and neighborhood of .

(9.22) **Example:** Let be indiscrete metric space, and . Calculate .

**Solution:** (1) if .

(2) if , then we have

a. if contains one element .

b. if contains more than one element .

3. if .

a. if contains one element .

b. if contains more than one element .

(9.23) **Theorem:** Let be a metric space, and , then we have

1. If .
2. .
3. .

**Proof:** (1) let .

Since

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(9.24) **Note:** Not necessary that , for example

Let be indiscrete metric space, if .

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(9.25) **Definition:** Let be a metric space, and . We said that is a boundary point of set , if . The set of all boundary points denoted by

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(9.26) **Theorem:** Let be a metric space, and , then

1. .
2. .
3. is a closed set in .
4. .
5. .

**Proof:** (1) let open set in and

By same way we prove that .

(9.27) **Example:** Let be discrete metric space, and . Calculate .

**Solution:** since is a closed and is closed

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