Boundary Layer Meteorology

Chapter 5

Turbulent kinetic energy

- **Turbulence kinetic energy (TKE) is one of the most important variables in micrometeorology, because it is a measure of the intensity of turbulence.**
- **It is directly related to the transport of momentum, heat and moisture through the BL.**
- **The individual terms in the TKE budget equation describe physical processes that generate turbulence.**
- **The relative magnitude of these processes determines the ability of the flow to maintain turbulence or become turbulent, and thus indicates flow stability.**

The TKE budget equation

- **Recall that**

$$\mathrm{TKE}/\mathrm{m} = \bar{e} = \tfrac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$

- **Summation notation ⇒**

$$\bar{e} = \tfrac{1}{2}(\overline{u_i'^2})$$

- **⇒ TKE/m is the summed velocity variance divided by 2.**
- **∴ start with the prognostic equation for the sum of the velocity variances and divide by 2 ⇒**

$$\frac{\partial \bar{e}}{\partial t} + \bar{u}_j \frac{\partial \bar{e}}{\partial x_j} = \delta_{i3} \frac{g}{\bar{\theta}_v} \overline{u_i'\theta_v'} - \overline{u_i'u_j'} \frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial (\overline{u_j'e})}{\partial x_j} - \frac{1}{\bar{\rho}} \frac{\partial \overline{u_i'p'}}{\partial x_i} - \varepsilon$$

I II III IV V VI VII

Term I represents the rate-of-change of TKE

Term II is the advection of TKE by the mean wind

Term III is the buoyant production or consumption term

Term IV is the mechanical or shear production term

Term V is the turbulent transport of TKE. It describes how TKE is moved around by the turbulent eddies

Term VI is the pressure correlation term that describes how TKE is redistributed by pressure perturbations.

Term VII represents the viscous dissipation of TKE, i.e. the conversion of TKE into heat.

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Choose a coordinate system aligned with the mean wind, assume horizontal homogeneity, and neglect subsidence ⇒

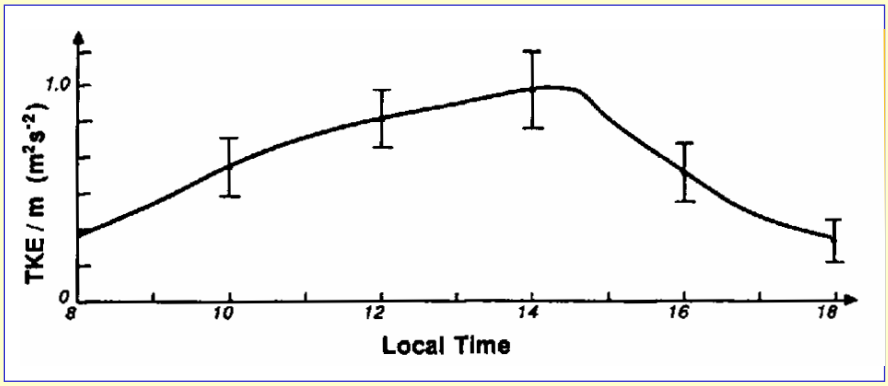
$$\frac{\partial \overline{e}}{\partial t} = \frac{g}{\overline{\theta}_v}\overline{w'\theta'_v} - \overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \frac{\partial(\overline{w'e})}{\partial z} - \frac{1}{\overline{\rho}}\frac{\partial \overline{w'p'}}{\partial z} - \varepsilon$$

I III IV V VI VII

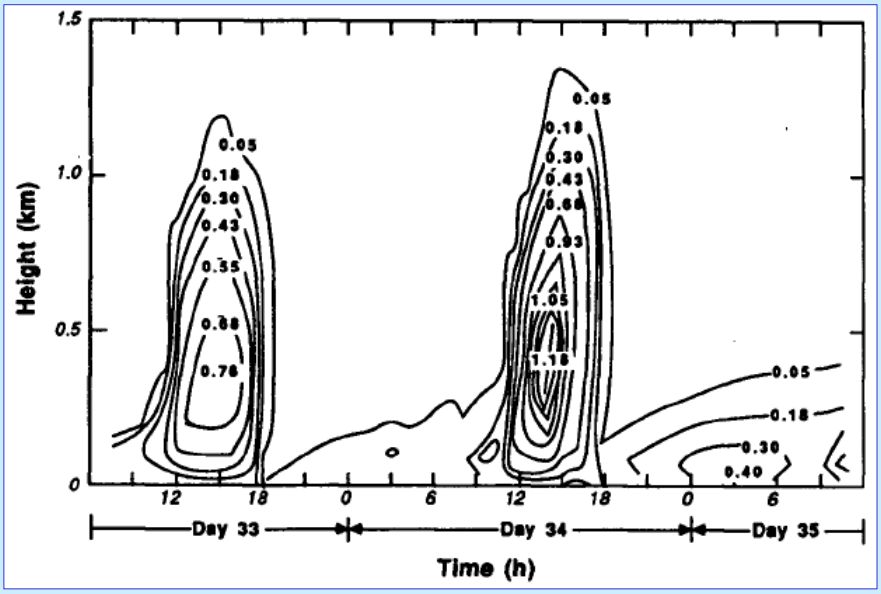
- **Turbulence is dissipative. Term VII is a loss term that always exists when TKE is nonzero.**
- **Physically: turbulence will tend to decrease with time unless it can be generated locally or transported there.**
- **Thus TKE is not a conserved quantity.**
- **The BL can be turbulent only if there are specific physical processes generating the turbulence.**

Contributions to the TKE budget

- **Term I: temporal variation**

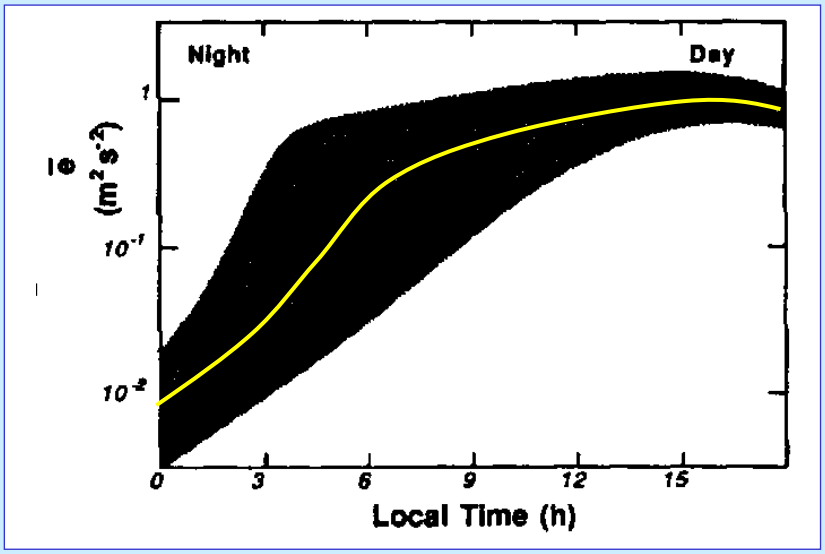


A typical daytime variation of TKE in convective conditions.



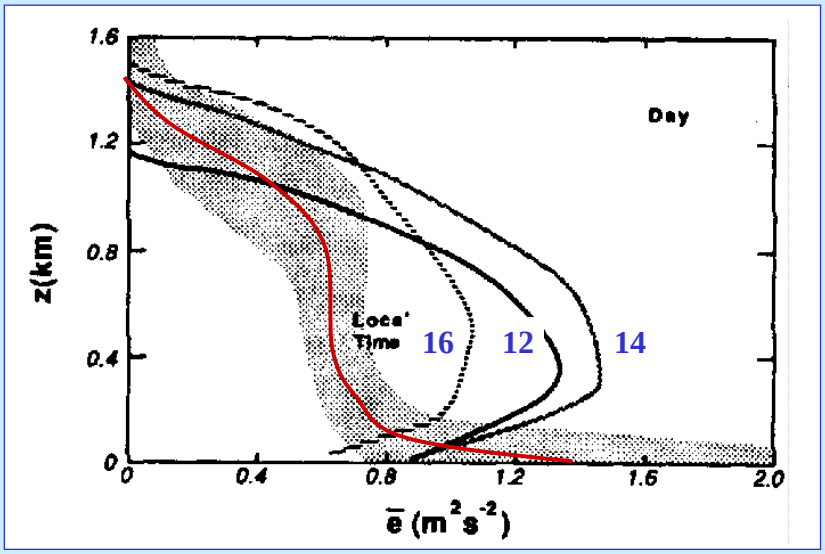
Modelled time and space variation of TKE (units m^2s^{-2}), for Wangara.

From Yamada & Mellor (1975)



Sample diurnal variation of observed range of TKE in the surface layer during November.

From Louis et al. (1983)



Lines show modelled vertical profiles of TKE during Day 33, Wangara. The shaded profile applies when both shear and buoyancy are active.

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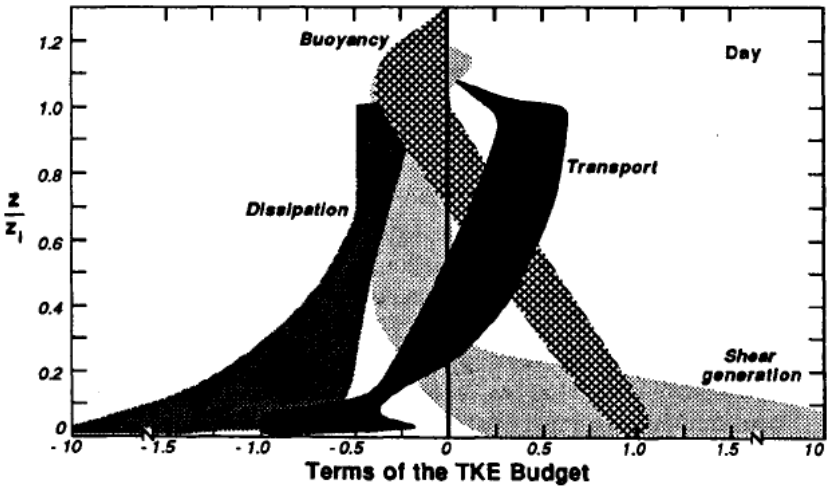

Normalized terms in the turbulence kinetic energy equation. The shaded areas indicate ranges of values. All terms are divided by w_*^3/z_i, which is on the order of 10^{-3} m^2s^{-3}.

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Advection

- **Term II: Advection**
- **Little is known about this term.**
- **When averaged over a horizontal area larger than about 10 km × 10 km, it is often assumed that there is little horizontal variation in TKE and that the advection term is negligible.**
- **This is probably a good assumption over most land surfaces.**
- **On a smaller scale, however, this term must be important.**

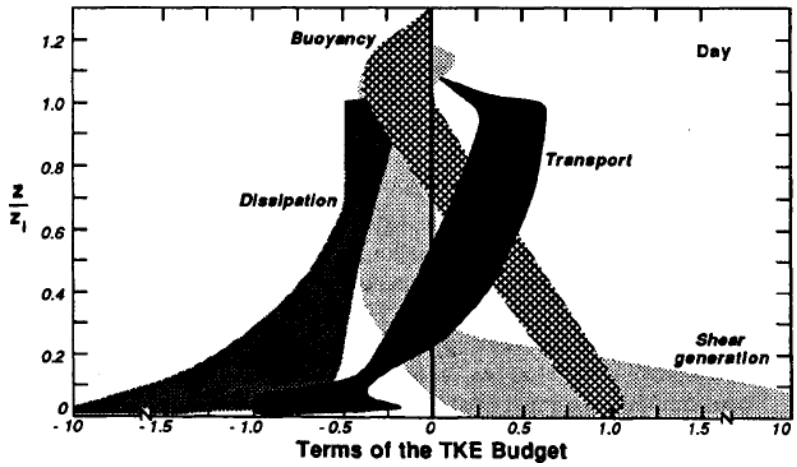
Example

Advection

TKE advected

- **Imagine a reservoir of water cooler than the surrounding land. The lack of heating over the reservoir would allow turbulence to decay in the overlying air, while air over the adjacent land surfaces could be in a state of active convection. A mean wind advecting air across the shores of this reservoir would cause significant change in the TKE budget.**
- **Over ocean surfaces, the advection term would probably be negligible even on the small scales.**

Buoyant production/ consumption



Normalized terms in the turbulence kinetic energy equation. The shaded areas indicate ranges of values. Based on data and models.

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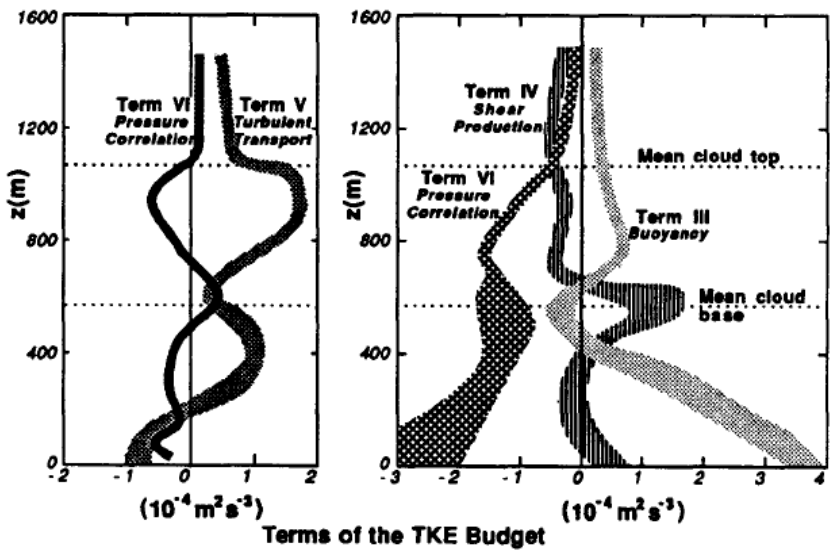
- **Term III Because this term is so important on days of free convection, it is often used to normalize all other terms.**
- **At the surface, Term III =** w_*^3 / z_i .

$$\frac{z_i}{w_*^3}\frac{\partial \overline{e}}{\partial t} = \frac{g z_i}{w_*^3 \overline{\theta}_v}\overline{w'\theta'_v} - \frac{z_i \overline{u'w'}}{w_*^3}\frac{\partial \overline{u}}{\partial z} - \frac{z_i}{w_*^3}\frac{\partial(\overline{w'e})}{\partial z} - \frac{z_i}{w_*^3}\frac{1}{\overline{\rho}}\frac{\partial \overline{w'p'}}{\partial z} - \frac{z_i \varepsilon}{w_*^3}$$

I III IV V VI VII

- **By definition, term III is unity at the surface.**

As is evident in (4.3.1j), the buoyancy term acts only on the vertical component of TKE. Hence, this production term is ***anisotropic*** (i.e., not isotropic). The return-to-isotropy terms of (4.3.1h-j) are responsible for moving some of the vertical kinetic energy into the horizontal directions. Again, the anisotropic nature of Term III confirms our picture of strong up and downdrafts within thermals.



Range of terms in the turbulence kinetic energy budget for a cloud-topped tropical boundary layer. The The transport term is split into the pressure correlation (PC) and turbulent transport (T) parts.

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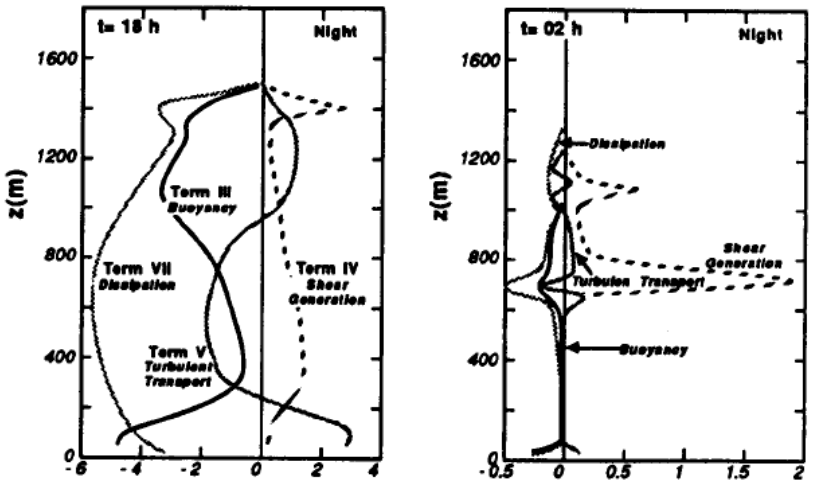
Consumption

- In statically stable conditions, an air parcel displaced vertically by turbulence would experience a buoyancy force pushing it back to its starting height.
- In this case, static stability tends to suppress, or consume, TKE and is associated with a negative value of Term III.
- Such conditions are present in the stable boundary layer at night over land, or anytime the surface is colder than the overlying air.

Example



Terms of the TKE Budget

Modelled turbulence kinetic energy budgets at t = 18 h and t = 02 h during the night 33-34, Wangara.

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Mechanical (shear) production

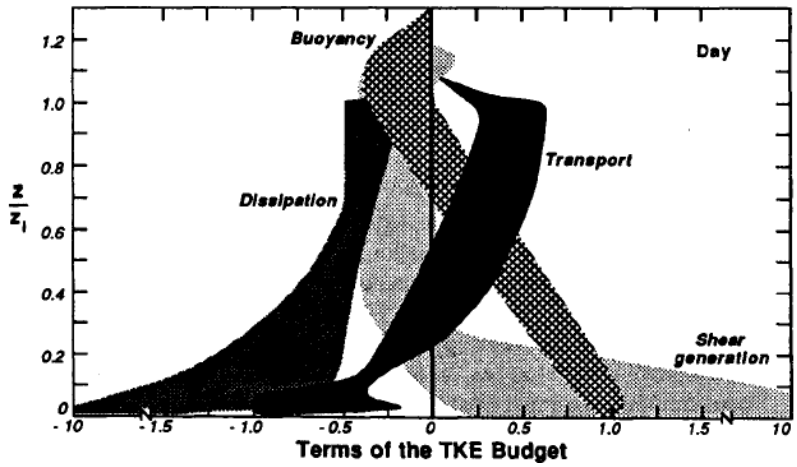
- **Term IV**

$$\frac{\partial \overline{e}}{\partial t} = \frac{g}{\overline{\theta}_v}\overline{w'\theta'_v} - \overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \frac{\partial(\overline{w'e})}{\partial z} - \frac{1}{\overline{\rho}}\frac{\partial \overline{w'p'}}{\partial z} - \varepsilon$$

I III IV V VI VII

- **When there is a turbulent momentum flux in the presence of a mean wind shear, the interaction between the two tends to generate more turbulence.**
- **Despite the negative term in Term IV, the momentum flux is usually of opposite sign to the mean shear, resulting in production, not loss, of turbulence.**

Buoyant production/ consumption



Normalized terms in the turbulence kinetic energy equation. The shaded areas indicate ranges of values. Based on data and models.

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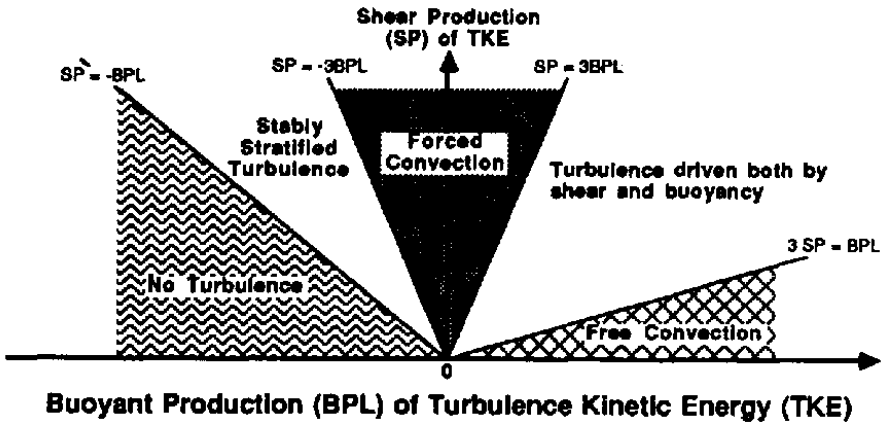
Shear versus buoyant production



Approximate regimes of free and forced convection.

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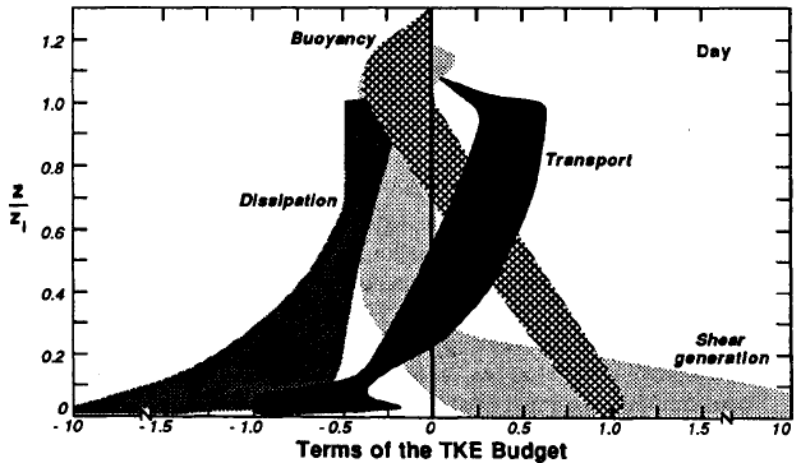
- **Magnitudes of the shear production term in surface layer are obviously greatest on a windy day, and are small on a calm day.**
- **In synoptic-scale cyclones the strong winds and overcast skies suggest that forced convection is applicable.**
- **On many days, turbulence is neither in a state of free nor forced convection because both the shear and buoyancy terms are contributing to the production of turbulence.**
- **At night over land, or anytime the ground is colder than the air, the shear term is often the only term that generates turbulence.**

Buoyant production/ consumption



Normalized terms in the turbulence kinetic energy equation. The shaded areas indicate ranges of values. Based on data and models.

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- **The greatest shears are associated with the change of** u **and** v **components of mean wind with height.**
- **Except in thunderstorms, the shear of** w **is negligible in the BL.**
- **From the equations for the components of variance, the shear production is greatest in the** x **and** y **components of TKE. Hence, shear production is also an anisotropic forcing - strongest in the horizontal.**
- **Both the buoyant and shear production terms can generate anisotropic turbulence.**
- **The difference is that shear generation produces turbulence primarily in the horizontal directions, while buoyant generation produces it primarily in the vertical.**
- **These differences are evident in the next figure ⇒**

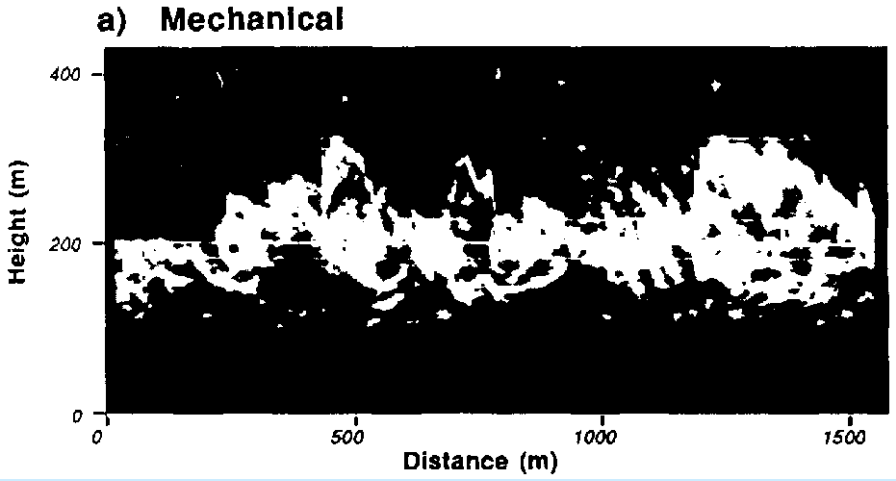

Fig. 5.8 Radar images of turbulence near the boundary layer top, showing (a) forced convection, and (b) free convection. After Noonkester (1974).

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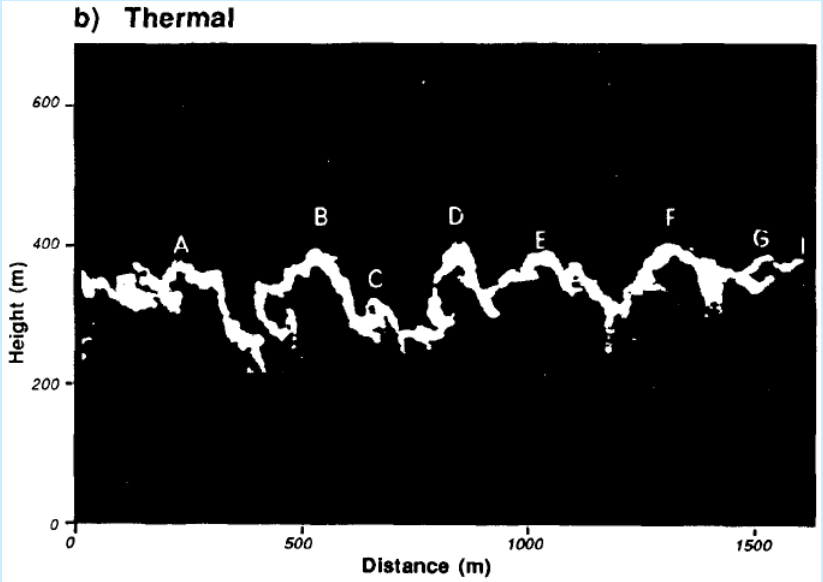

Fig. 5.8 Radar images of turbulence near the boundary layer top, showing (a) forced convection, and (b) free convection. After Noonkester (1974).

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Turbulent transport

- **Term V**

V

$$\frac{\partial \bar{e}}{\partial t} = \frac{g}{\bar{\theta}_v}\overline{w'\theta_v'} - \overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \frac{\partial(\overline{w'e})}{\partial z} - \frac{1}{\bar{\rho}}\frac{\partial \overline{w'p'}}{\partial z} - \varepsilon$$

- $\overline{w'e}$ **represents the vertical turbulent flux of TKE.**
- **As for other vertical fluxes, the change in flux with heights is more important than magnitude of flux.**
- **Term V is a flux divergence term; if there is more flux into a layer than leaves, then the magnitude of TKE increases.**
- **This term simply redistributes TKE: when integrated over the depth of the mixed layer, it gives zero contribution.**

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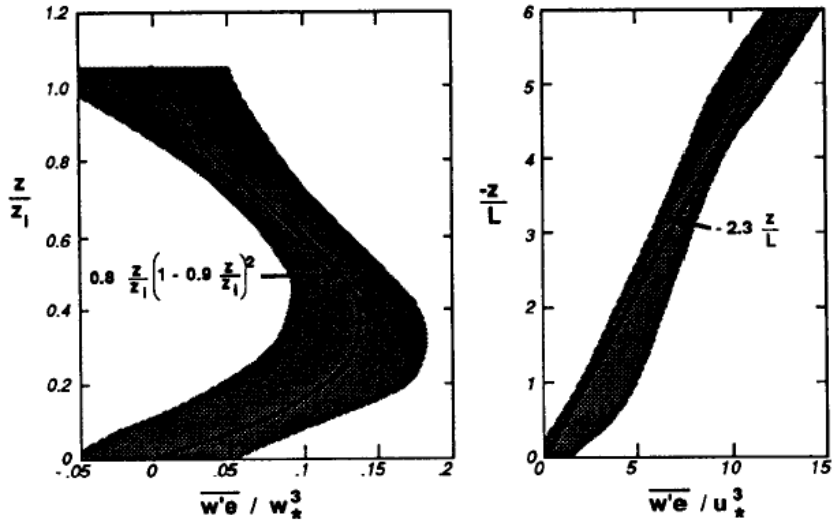

Range of vertical profiles of the normalized vertical flux of turbulence kinetic energy using mixed layer scaling (left) and surface-layer scaling (right) where L is the Obukhov length.

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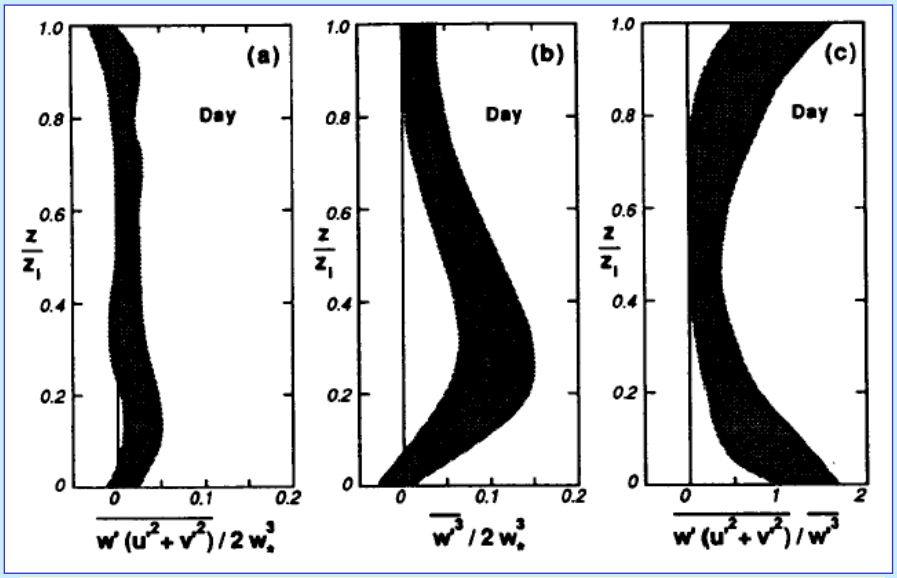


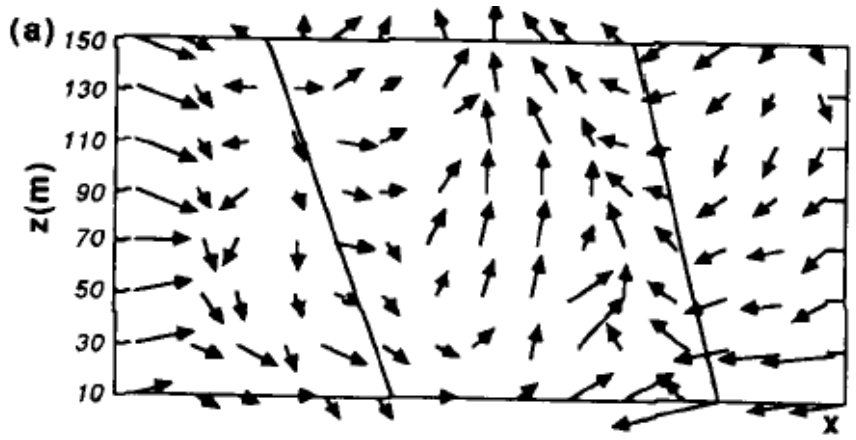
(a) Range of vertical profiles of the normalized vertical flux of horizontal variance; (b), the vertical flux of vertical variance; and (c), the ratio of the two during daytime.

Pressure correlation

- **Term VI** VI

$$\frac{\partial \overline{e}}{\partial t} = \frac{g}{\overline{\theta}_v}\overline{w'\theta'_v} - \overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \frac{\partial(\overline{w'e})}{\partial z} - \frac{1}{\overline{\rho}}\frac{\partial \overline{w'p'}}{\partial z} - \varepsilon$$

- **Turbulence: static pressure fluctuations are exceedingly difficult to measure in the atmosphere.**
- **The magnitudes of these fluctuations are very small, being on the order of 0.005 kPa (0.05 mb) in the convective surface layer to 0.001 kPa (0.01 mb) or less in the mixed layer.**
- **Pressure sensors with sufficient sensitivity to measure these static pressure fluctuations are contaminated by the large dynamic pressure fluctuations associated with turbulent and mean motions ⇒ correlations such as w'p' calculated from experimental data often contain more noise than signal.**



(a) Composite of measured circulation patterns in a vertical cross-section through convective thermals. Velocity vectors: deviations from the mean wind. Solid lines: the boundaries of the temperature ramp associated with a thermal updraft; they are separated by a physical distance on the order of 100 m.

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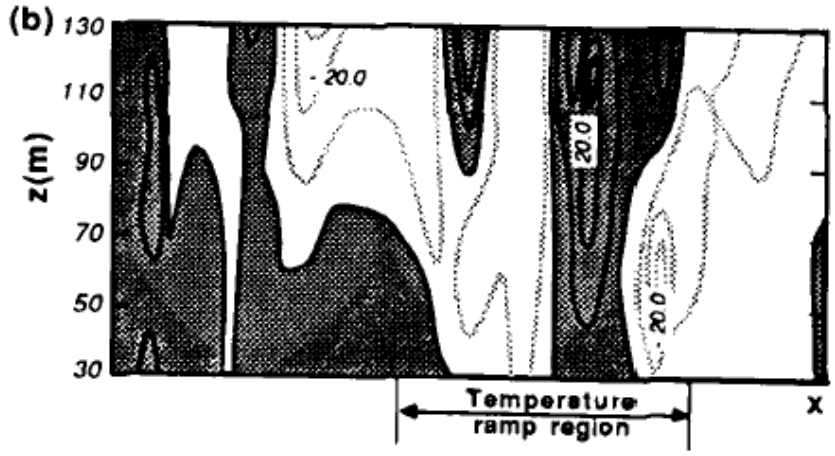
- **What little is known about the behaviour of pressure correlation terms is estimated as a residual in the budget equations discussed previously.**
- **If all of the other terms in a budget equation are measured or parameterized, then the residual necessary to make the equation balance includes an estimate of the unknown term(s) together with the accumulated errors.**
- **An obvious hazard of this approach is that the accumulated errors from all of the other terms can be quite large.**
- **Estimates of $\overline{w'p'}$ in the surface layer are shown in the next figure using this method, composited with respect to a large number of convective plume structures.**



(b) Contour z-z plot of $\overline{w'p'}/\rho u_*^3$, where the horizontal axis represents a composite of many thermals. Contour interval is 10.0.

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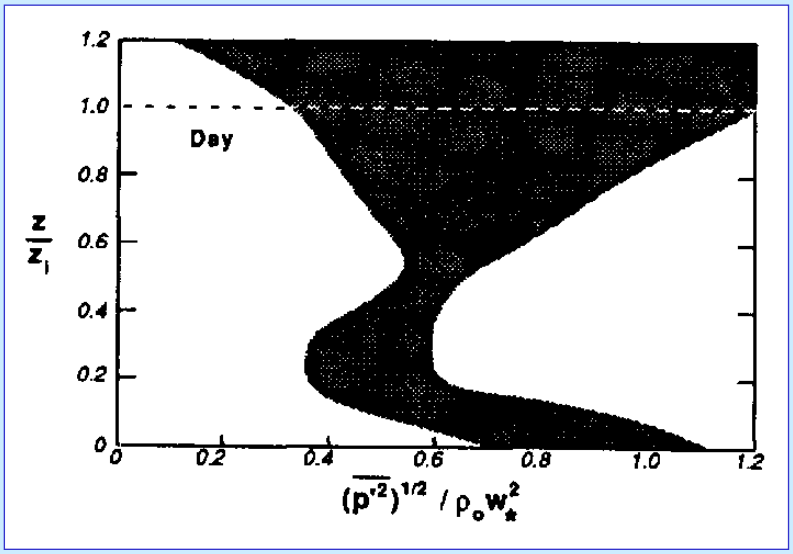

Normalized Doppler-radar derived standard deviation of perturbation pressure.

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Waves

- **Perturbation from a mean can describe waves as well as turbulence.**
- **Given measured values of $\overline{w'p'}$, it is impossible to separate the wave and turbulence contributions without additional information.**
- **Work in linear gravity wave theory shows that $\overline{w'p'}$ is equal to the upward flux of wave energy for a vertically propagating internal gravity wave within a statically stable environment.**
- **This suggest that turbulence energy can be lost from the mixed layer top in the form of internal gravity waves being excited by thermals penetrating the stable layer at the top of the mixed layer.**

- **The amount of energy lost by gravity waves may be on the order of less than 10% of the total rate of TKE dissipation, but the resulting waves can sometimes enhance or trigger clouds.**
- **Turbulence within stable nocturnal boundary layers can also be lost in the form of waves.**
- **One concludes that the pressure correlation term not only acts to redistribute TKE within the BL, but it can also drain energy out of the BL.**

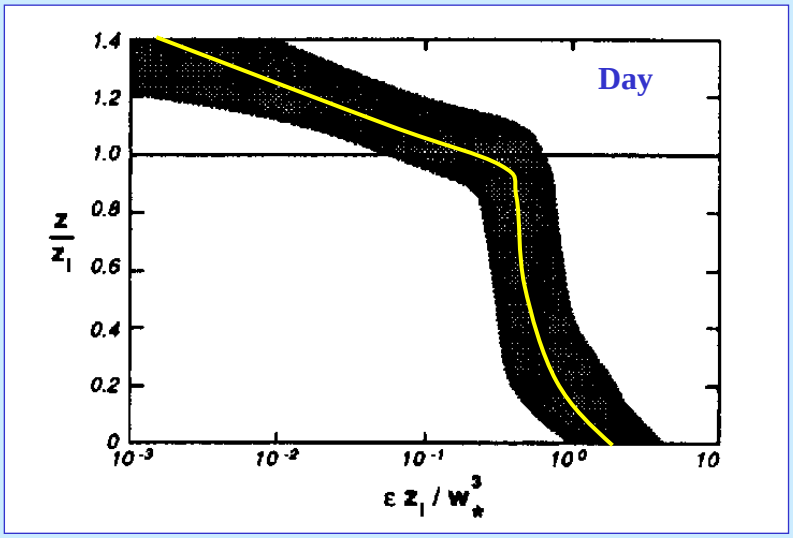
Dissipation

- **Term VII**

$$\frac{\partial \overline{e}}{\partial t} = \frac{g}{\overline{\theta}_v}\overline{w'\theta'_v} - \overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \frac{\partial(\overline{w'e})}{\partial z} - \frac{1}{\overline{\rho}}\frac{\partial \overline{w'p'}}{\partial z} - \varepsilon$$

VII

- **The molecular destruction of turbulent motions is greatest for the smallest size eddies.**
- **The more intense this small-scale turbulence, the greater the rate of dissipation.**
- **Small-scale turbulence is, in turn, driven by the cascade of energy from the larger scales.**
- **Typical profiles are shown in the next figures ⇒**



Range of normalized dissipation rate (ε) profiles during the day-time, where z_i is the mixed layer depth and w_* is the convective velocity scale.

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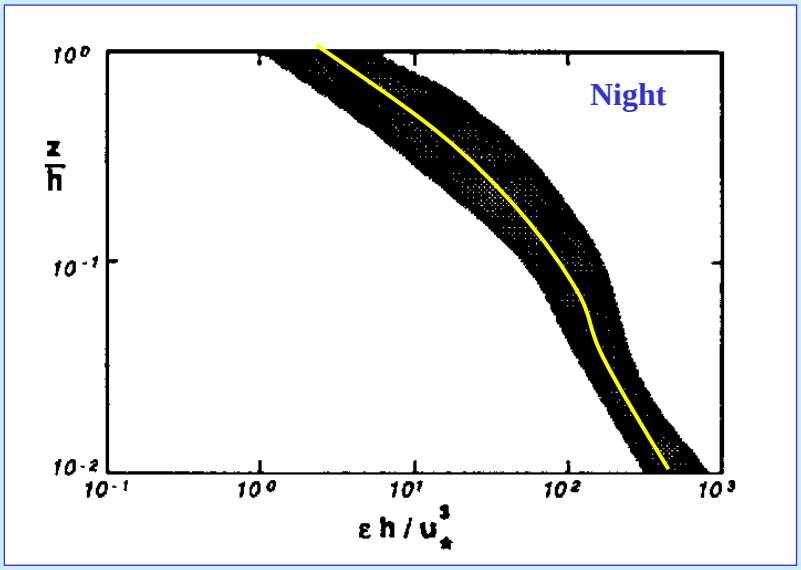

Range of normalized dissipation rate (ε) profiles at night, where h is boundary layer depth and u_* is the friction velocity.

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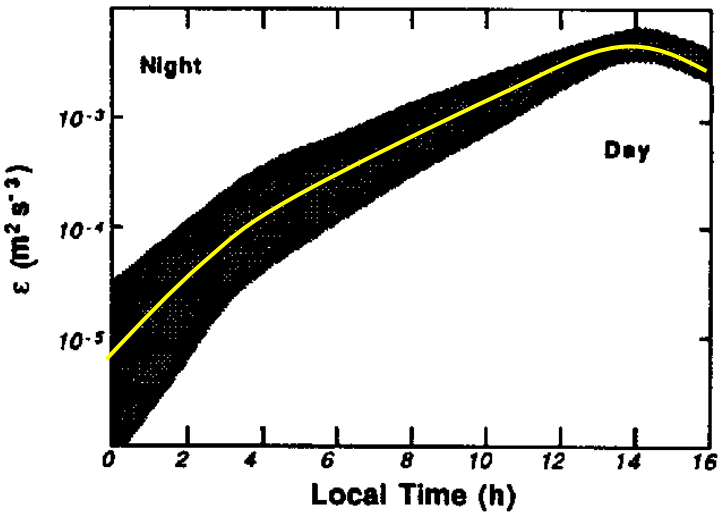
Dissipation

- Because turbulence is not conserved, the greatest TKEs, and hence greatest dissipation rates, are frequently found where TKE production is the largest - near the surface.
- However, the dissipation rate is not expected to perfectly balance the production rate because of the various transport terms in the TKE budget.
- The close relationship between TKE production rate, intensity of turbulence, and dissipation rate is shown in the next figure ⇒



Example of variation of dissipation rate with time from night to day.

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Example

- **Problem: At a height of** $z = 300$ **m in a** 1000 **m thick mixed layer the following conditions were observed:**

  $\partial \overline{U} / \partial z = 0.01$ **s**$^{-1}$,

  $\theta_v = 25$**°C,**

  $\overline{w'\theta_v'} = 0.15$ **K m s**$^{-1}$**, and**

  $\overline{u'w'} = -0.03$ **m**^{2}**s**$^{-2}$**.**

  Also, the surface virtual heat flux is 0.24 **K m s**$^{-1}$**.**

- **If the pressure and turbulent transports are neglected, then (a) what dissipation rate is required to maintain a locally steady state at** $z = 300$ **m; and (b) what are the values of the normalized TKE terms?**

- **Solution: (a) Since no information was given about the** V **component of velocity or stress, we assume that the** x**-axis is aligned with the mean wind.**

- **Consider the TKE budget:**

$$\cancel{\frac{\partial \overline{e}}{\partial t}} = \frac{g}{\overline{\theta}_v}\overline{w'\theta_v'} - \overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \cancel{\frac{\partial(\overline{w'e})}{\partial z}} - \frac{1}{\overline{\rho}}\cancel{\frac{\partial \overline{w'p'}}{\partial z}} - \varepsilon$$

= 0 steady **= 0** **= 0** **---- given ----**

$$\Rightarrow \quad \varepsilon = \frac{g}{\overline{\theta}_v}\overline{w'\theta_v'} - \overline{u'w'}\frac{\partial \overline{u}}{\partial z}$$

$$\Rightarrow \quad \varepsilon = \frac{9.8}{288.15} \times 0.15 - (-0.03) \times 0.01 = 5.23 \times 10^{-3} \text{ m}^2 \text{ s}^{-2}$$

- **Solution: (b) To normalize the equations, calculate**

$$\frac{w_*^3}{z_i} = \frac{g}{\overline{\theta}_v}\overline{w'\theta'_v} = 7.89\times10^{-3}\ m^2\,s^{-3}$$

- **Divide all terms by this value and rewrite in the same order:**

$$\cancel{\frac{\partial \overline{e}}{\partial t}} = \frac{g}{\overline{\theta}_v}\overline{w'\theta'_v} - \overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \cancel{\frac{\partial(\overline{w'e})}{\partial z}} - \frac{1}{\overline{\rho}}\cancel{\frac{\partial \overline{w'p'}}{\partial z}} - \varepsilon$$

0 = 0.625 + 0.038 - 0 - 0 - 0.663

Special problems for turbulent flow

- **In principle, the equations can be applied directly to turbulent flows, but this is generally too complicated.**
- **We would not be able to resolve all turbulent scales down to the smallest eddy to determine the initial condition.**
- **Instead, for simplicity, we pick some cut-off eddy size below which we include only the statistical effects of turbulence.**
- **In some mesoscale and synoptic scale models the cut off is on the order of 10 to 100 km, while for some boundary layer models known as large eddy simulation models, the cut off is on the order of 100 m.**

Mean KE and its interaction with turbulence

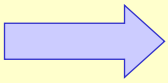
- **TKE equation**

$$\frac{\partial \overline{e}}{\partial t} + \overline{u}_j \frac{\partial \overline{e}}{\partial x_j} = \delta_{i3} \frac{g}{\overline{\theta}_v} \overline{u_i' \theta_v'} - \overline{u_i' u_j'} \frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial (\overline{u_j' e})}{\partial x_j} - \frac{1}{\overline{\rho}} \frac{\partial \overline{u_i' p'}}{\partial x_i} - \varepsilon$$

I II III IV V VI VII

Term IV is the mechanical or shear production term.

- **We would expect that the production of TKE is accompanied by a corresponding loss of KE from the mean flow.**
- **Let us derive an equation for the mean flow KE: multiply the equation for the mean wind by $\overline{u}_i$.**



- **Mean KE equation**

III IV zero

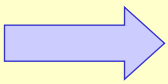
$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\overline{u}_i^2\right) + \overline{u}_j \frac{\partial}{\partial x_j}\left(\tfrac{1}{2}\overline{u}_i^2\right) = -g\delta_{i3}\overline{u}_i + f\varepsilon_{ij3}\overline{u}_i\overline{u}_j$$

$$- \frac{\overline{u}_i}{\overline{\rho}} \frac{\partial \overline{p}}{\partial x_i} + \nu \overline{u}_i \frac{\partial \overline{u}_i}{\partial x_j^2} - \overline{u}_i \frac{\partial \overline{u_i' u_j'}}{\partial x_j}$$

V VI VII

Term VII $\Rightarrow$

$$-\overline{u}_i \frac{\partial \overline{u_i' u_j'}}{\partial x_j} = \overline{u_i' u_j'} \frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial \left(\overline{u_i' u_j'} \overline{u}_i\right)}{\partial x_j}$$



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TKE equation

$$\frac{\partial \overline{e}}{\partial t} + \overline{u}_j \frac{\partial \overline{e}}{\partial x_j} = \delta_{i3} \frac{g}{\overline{\theta}_v} \overline{u_i' \theta_v'} - \overline{u_i' u_j'} \frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial (\overline{u_j' e})}{\partial x_j} - \frac{1}{\overline{\rho}} \frac{\partial \overline{u_i' p'}}{\partial x_i} - \varepsilon$$

Mean KE equation

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\overline{u}_i^2\right) + \overline{u}_j \frac{\partial}{\partial x_j}\left(\tfrac{1}{2}\overline{u}_i^2\right) = -g\overline{w} - \frac{\overline{u}_i}{\overline{\rho}} \frac{\partial \overline{p}}{\partial x_i} +$$

$$\nu \overline{u}_i \frac{\partial \overline{u}_i}{\partial x_j^2} - \frac{\partial \left(\overline{u_i' u_j'} \overline{u}_i\right)}{\partial x_j} + \overline{u_i' u_j'} \frac{\partial \overline{u}_i}{\partial x_j}$$

- **Energy that is mechanically produced as turbulence is lost from the mean flow and vice versa.**