ESCI 485 – Air/sea Interaction
Lesson 3 – The Surface Layer

References: *Air-sea Interaction: Laws and Mechanisms*, Csanady
Structure of the Atmospheric Boundary Layer, Sorbjan

THE PLANETARY BOUNDARY LAYER

- **The atmospheric planetary boundary layer (PBL) is that region of the atmosphere in which turbulent fluxes are not negligible. Its depth can vary depending on the stability of the atmosphere.**
  - **In deep convection the PBL can be of the same order as the entire troposphere.**
  - **Usually it is of the order of 1 km or so.**
- **The PBL can be further broken down into three layers**
  - **The mixed layer – the upper 90% or so of the PBL in which eddies have nearly mixed properties such as potential temperature, momentum, and moisture.**
  - **The surface layer – The lowest 10% or so of the PBL in which the turbulent fluxes dominate all other terms (Coriolis and PGF) in the momentum equations.**
  - **The viscous sub-layer – A very thin layer (order of cm or less) right near the surface where viscous effects may be important.**

THE SURFACE LAYER

- **In the surface layer the turbulent momentum flux dominates all other terms in the momentum equations.**
- **The surface layer extends upwards on the order of 10 m or so.**
- **The Reynolds fluxes (stresses) in the surface layer can be assumed constant with height, and have the same magnitude as the interfacial stress (the stress between the air and water).**
- **In the surface layer, the eddy length scale is proportional to the height, z, above the surface.**

- Based on the *similarity principle*, the following dimensionless group is formed from the friction velocity, eddy length scale, and the mean wind shear

$$\frac{z}{u*}\frac{dU}{dz} = B, \tag{1}$$

  where B is a dimensionless constant.

- Integrating Eq. (1) gives a logarithmic wind profile

$$U(z) = Bu* \ln z + C \tag{2}$$

  here B and C are constants.

- If z_0 is defined as the level where the mean wind speed is zero (called the *roughness length*), then C has the value of

$$C = -Bu* \ln z_0 \tag{3}$$

  and we have

$$U(z) = Bu* \ln \left(z/z_0 \right). \tag{4}$$

- The constant B is usually written (for some obscure reason) as

$$B = 1/\kappa \tag{5}$$

  where κ is the von Karman constant, and has an empirical value of 0.4. Thus, the log-wind profile is normally written as

$$U(z) = \left(u*/\kappa \right) \ln \left(z/z_0 \right). \tag{6}$$

- Another way of writing the log-wind profile is to use an arbitrary reference level, $z = r$, and to call the velocity at this reference level C. Then the log-wind profile is

$$U(z) = \left(u*/\kappa \right) \ln \left(z/r \right) + C, \tag{7}$$

  where C and r are empirical constants that are found by observation.

WIND WAVES

- A steady wind blowing over water will generate wind waves.
- Up to a point, the longer the wind blows, the higher the waves will become. After a certain period of time the wave height will be steady.

- **If the wind is blowing offshore, wave height will increase downwind up to a point, after which the wave height will be steady. (The distance over which the wind is blowing is called the *fetch*.)**
- **Wave height is defined as the vertical distance from a trough to a crest (twice the amplitude).**
- **The significant wave height is defined to be the average height of the highest 33% of the waves, and is denoted at $H_{1/3}$.**
- **The restoring force for the longer waves is gravity, so they are called *gravity waves*.**
- **The restoring force for the very shortest waves ($\lambda \sim$ cm) is surface tension, and they are called *capillary waves*.**
- **We would expect that the significant wave height would depend only on the wind speed and gravity. Therefore, using dimensional analysis, we expect that**

$$H_{1/3} \propto u*^2 \big/ g , \qquad\qquad (8)$$

so that the combination $u*^2/g$ can be thought of as a characteristic wave height scale.
- **The phase speed C_p is the speed at which an individual trough or crest advances.**
- **The group velocity, C_g, is the speed at which the energy of the wave propagates.**
  - **The group velocity is not usually the same as the phase speed. Therefore, the energy of the wave propagates at a velocity different than that of what your eye sees following an individual crest. We call such waves *dispersive*.**
- **For most water waves, the group velocity is less than the phase speed, so that individual crests seem to move "through" the packet of wave energy. New crests seem to be created at the trailing edge, move through the packet of energy, and disappear at the leading edge.**
- **For very long wavelengths, (i.e., the wavelength is larger than the depth of the water) the phase speed and group velocity are the same, and the waves are said to be non-dispersive.**
- **For very short wavelengths (capillary waves), where surface tension is the restoring force, the group velocity is actually greater than the phase speed.**
- **Waves enhance momentum transfer in several ways**

- o **The waves increase the roughness of the surface, providing more interaction with the air.**
- o **The wave faces allow horizontal pressure to act on the water.**
- o **As the waves break, the turbulent motion mixes momentum deeper into the water.**
- **Evidence suggests that it is the shortest waves that are most efficient and important for air-sea momentum transfer, because they are steep and prone to breaking.**

CHARNOCK'S LAW

- **Observations by Charnock in 1955 over a reservoir established that the parameter *r* in Eq. (7) is related to the friction velocity and gravity via**

$$r = u^{*2}/g .$$
(9)

This makes sense, since this is also a wave height scale, and we would expect wave height to factor into Eq. (7) somehow. Using Eq. (9) in Eq. (7) yields

$$U(z) = \frac{u^*}{\kappa} \ln\left(\frac{g z}{u^{*2}}\right) + C .$$
(10)

- o **Charnock also estimated that the constant *C* has a value of about 12.5.**
- o **Equation (10) is known as Charnock's Law.**
- **Since u^{*2}/g is a characteristic wave height scale (see previous section), Charnock's law says that the wind speed at a given level will decrease as wave height increases. This makes sense, since a higher wave height indicates a rougher surface.**
- **It is customary to use a height of $z = h = 10$ m when measuring and reporting wind data. Therefore, Charnock's law becomes**

$$U(h) = \frac{u^*}{\kappa} \ln\left(\frac{g h}{u^{*2}}\right) + C .$$
(11)

DRAG COEFFICIENT

- **What we would ultimately like is to somehow parameterize the friction velocity in terms of something we can easily measure, such as the wind as a certain height above the water, $U(h)$. We can do this by rewriting Charnock's law as**

$$u*^2 = \frac{[U(h)]^2}{\left[\frac{1}{\kappa}\ln\left(\frac{gh}{u*^2}\right) + C\right]^2} = C_D[U(h)]^2,$$ (12)

where C_D is the *drag coefficient*, and is

$$C_D \equiv \left[\frac{1}{\kappa}\ln\left(\frac{gh}{u*^2}\right) + C\right]^{-2}.$$ (13)

- **Notice that the drag coefficient itself is a function of $u*^2$. In reality, the drag coefficient is determined empirically, rather than through the use of the above formula, and is a function of wind speed, $U(h)$.**

THE NON-NEUTRAL SURFACE LAYER

- **Charnock's law assumes that the properties of turbulence in the surface layer depend only on friction velocity and height, $u*$ and z, and that the wave height only depends on friction velocity and gravity, $u*$ and g. Implicit in these assumptions are that the surface layer is *neutrally stable*, meaning it is neither stable nor unstable.**

- **If the surface layer is stable, then vertical mixing is inhibited. This would be expected to reduce momentum transfer, and alter the mean wind profile by decreasing the winds at the lowest levels.**

- **If the surface layer is unstable, then vertical mixing is enhanced, and turbulent momentum transfer would also be expected to be enhanced, increasing the winds at the lowest levels.**

- **We therefore need an additional parameter to characterize the stability of the surface layer, and this is provided by the *buoyancy flux*, B_0.**

- **The perturbation buoyancy is defined as**

$$b' = -g\frac{\rho'}{\bar{\rho}} = g\frac{T_v'}{\overline{T_v}},$$ (14)

where ρ' is the departure of density and T'_v is the departure from the average virtual temperature $[T_v = T(1 + 0.61q)]$.

● Through some tedious manipulation (shown below) we get (because $q \ll 1$)

$$\frac{T'_v}{\overline{T}_v} = \frac{T - \overline{T}_v}{\overline{T}_v} = \frac{T_v}{\overline{T}_v} - 1 = \frac{T(1 + 0.61q)}{\overline{T}(1 + 0.61\overline{q})} - 1 \approx \frac{T}{\overline{T}}(1 + 0.61q)(1 - 0.61\overline{q}) - 1$$

$$\approx \frac{T}{\overline{T}}(1 + 0.61q - 0.61\overline{q}) - 1 = \frac{T}{\overline{T}}(1 + 0.61q') - 1 = \frac{T - \overline{T} + 0.61Tq'}{\overline{T}} \approx \frac{T'}{\overline{T}} + 0.61q'$$

so that the buoyancy becomes

$$b' = g\frac{T'}{\overline{T}} + 0.61gq'. \tag{15}$$

● Buoyancy accounts for both the effects of heat and moisture on the stability of the atmosphere. Warmer temperature and higher humidity increase the buoyancy.

● The buoyancy itself can be thought of as a property that can be transported. If buoyancy is being added to the surface layer, than turbulence will be enhanced. If buoyancy is taken from the surface layer, than turbulence will be decreased.

● There is a vertical flux of buoyancy at the interface between the air and water. The Reynolds-averaged buoyancy flux at the interface between the water and the air is

$$B_0 = \left(\overline{w'b'}\right)_0. \tag{16}$$

WIND PROFILE IN THE NON-NEUTRAL SURFACE LAYER

● To include the buoyancy flux as an additional parameter needed to characterize the flow in the non-neutral surface layer we define a new length scale, called the *Obukhov length* (*L*), as

$$L \equiv -\frac{u*^3}{\kappa B_0}. \tag{17}$$

o The Obukhov length can be interpreted physically as the height above the surface when buoyancy first dominates over mechanical production of turbulence.

o The Obukhov length is positive or a stable surface layer.

- o **The Obukhov length is infinite for a neutral surface layer, since in a neutral surface layer there is never a height where buoyancy dominates.**
- o **The Obukhov length is negative for an unstable surface layer, since buoyancy dominates all the time, soyou would have to go to some negative (virtual) altitude below the water to reach a point where buoyancy dominates (physically ridiculous, but it keeps the mathematics consistent).**
- **The wind profile in the neutral surface layer depended upon the friction velocity, eddy length scale, and the mean wind shear, which gave us**

$$\frac{z}{u*}\frac{dU}{dz}=\frac{1}{\kappa}. \tag{18}$$

- **In the non-neutral surface layer we must also include the effects of stability, which we expect to be a function of both height and the Obukhov length. We therefore write**

$$\frac{z}{u*}\frac{dU}{dz}=\frac{1}{\kappa}\phi\left(\frac{z}{L}\right), \tag{19}$$

  where ϕ is the *stability function*, and is a dimensionless function of *z/L*.

- **The stability function is determined empirically. A commonly used form is**

$$\varphi\left(\frac{z}{L}\right)=1+\beta\frac{z}{L}; \qquad \textit{stable}$$
$$\varphi\left(\frac{z}{L}\right)=1 \qquad\qquad \textit{neutral} \tag{20}$$
$$\varphi\left(\frac{z}{L}\right)=\frac{1}{\sqrt[4]{(1-\alpha z/L)}}; \quad \textit{unstable}.$$

- **The figure below shows a qualitative graph of the stability parameter as a function of *z/L*.**
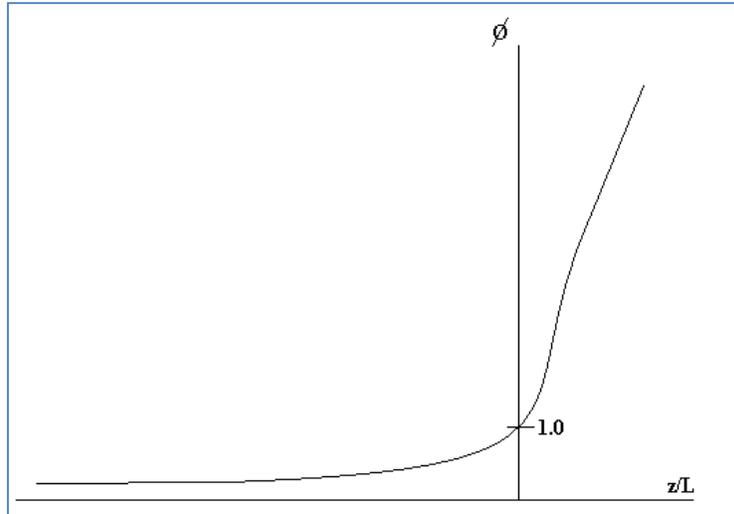
- **The stability function modifies the wind profile in the surface layer.**
  - o **For the stable surface layer the winds are decreased.**
  - o **For the unstable surface layer the winds are increased**
  - o **There is no change for a neutral layer, which remains logarithmic.**
- **The figure below shows qualitatively the wind profiles in the stable, neutral, and unstable surface layers.**
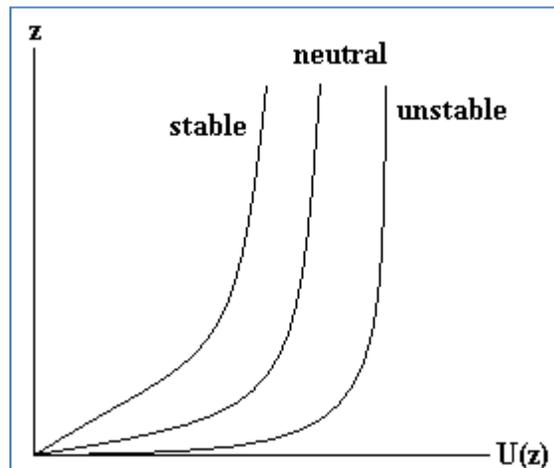
METHODS OF DETERMINING THE MOMENTUM FLUX

- **Determining the momentum flux in the surface layer can be achieved through either direct measurement of the turbulent momentum components, or by indirect methods. Some of these methods are described below.**
- *Profile method*
  - Use wind measurements at different levels to find the log-wind profile.
  - From log-wind profile the friction velocity and roughness length can be inferred.
  - Once the friction velocity is known the momentum flux is found via

    $$u* = \left(\left| \overline{w'u'} \right| \right)^{1/2}$$

  - This method is not particularly accurate, and if the surface layer is not neutral, then additional complications from the stability parameter arise.
- *Direct observation* of *u′* and *w′*
  - This method requires an average over long period of time (~ 1 hr), during which the velocity itself might change.
  - Instruments to measure velocity components must be aligned very precisely.
  - The mast or tower holding the instruments may interfere with wind flow and introduce errors.
- *Dissipation method*
  - Determine the energy spectrum of turbulence using Fourier transform.
  - Use the relation between the energy spectrum and dissipation to calculate the dissipation rate,

    $$\phi(k) = a\varepsilon^{2/3} k^{-2/3}. \tag{21}$$

  - Assume that in the surface layer the dissipation rate is related to the friction velocity via

    $$\varepsilon = u*^3 / \kappa z, \tag{22}$$

    so that the friction velocity is

    $$u* = \left(\kappa z \varepsilon \right)^{1/3}. \tag{23}$$