



# **Chapter Three**

## **Mathematical Operations with Arrays**

## Introduction

- Once variables are created in MATLAB they can be used in a wide variety of mathematical operations.
- Addition and subtraction are relatively simple operations.
- The other basic operations-multiplication, division, and exponentiation.

## LECTURE 3-1

### 3.1 Addition and Subtraction

- The operations, addition and subtraction can be used to add/subtract arrays of identical size and to add /subtract a scalar to an array.
- When two arrays are involved the sum, or the difference, of the arrays is obtained by adding, or subtracting, their corresponding elements.

If A and B are two arrays (for example, 2 x 3 matrices),

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \end{bmatrix}$$

Then the matrix that is obtained by adding A and B is:

$$\begin{bmatrix} (A_{11} + B_{11}) & (A_{12} + B_{12}) & (A_{13} + B_{13}) \\ (A_{21} + B_{21}) & (A_{22} + B_{22}) & (A_{23} + B_{23}) \end{bmatrix}$$

Examples are:

```
>> VectA=[8 5 4]; VectB=[10 2 7];
>> VectC=VectA+VectB
VectC =
    18     7    11
>> A=[5 -3 8; 9 2 10]
A =
     5     -3     8
     9     2    10
>> B=[10 7 4; -11 15 1]
B =
    10     7     4
   -11    15     1
>> A-B
ans =
    -5    -10     4
    20    -13     9
>> C=A+B
C =
    15     4    12
    -2    17    11
>> VectA+A
??? Error using ==> plus
Matrix dimensions must agree.
```

Define two vectors.

Define a vector VectC that is equal to VectA + VectB.

Define two 2x3 matrices A and B.

Subtracting matrix B from matrix A.

Define a matrix C that is equal to A + B.

Trying to add arrays of different size.

An error message is displayed.

- When a scalar (number) is added to/ subtracted from an array, the scalar is added to or subtracted from all the elements of the array. Examples are:

```

>> VectA=[1 5 8 -10 2]
VectA =
     1     5     8    -10     2
>> VectA+4
ans =
     5     9    12     -6
>> A=[6 21 -15; 0 -4 8]
A =
     6    21   -15
     0    -4     8
>> A-5
ans =
     1    16   -20
    -5    -9     3

```

Define a vector named VectA.

Add the scalar 4 to VectA.

4 is added to each element of VectA.

Define a 2 x 3 matrix A.

Subtract the scalar 5 from A.

5 is subtracted from each element of A.

### 3.2 Array Multiplication

The multiplication operation \* is executed by MATLAB according to the rules of linear algebra.

1. This means that if A and B are two matrices, the operation  $A * B$  can be carried out only if the number of columns in matrix A is equal to the number of rows in matrix B. The result is a matrix that has the same number of rows as A and the same number of columns as B. For example, if A is a 4 x 3 matrix and B is a 3 x 2 matrix:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \\ A_{41} & A_{42} & A_{43} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix}$$

then the matrix that is obtained with the operation  $A * B$  has dimensions 4 x 2 with the elements:

$$\begin{bmatrix} (A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31}) & (A_{11}B_{12} + A_{12}B_{22} + A_{13}B_{32}) \\ (A_{21}B_{11} + A_{22}B_{21} + A_{23}B_{31}) & (A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32}) \\ (A_{31}B_{11} + A_{32}B_{21} + A_{33}B_{31}) & (A_{31}B_{12} + A_{32}B_{22} + A_{33}B_{32}) \\ (A_{41}B_{11} + A_{42}B_{21} + A_{43}B_{31}) & (A_{41}B_{12} + A_{42}B_{22} + A_{43}B_{32}) \end{bmatrix}$$

A numerical example is:

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 6 & 1 \\ 5 & 2 & 8 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 1 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} (1 \cdot 5 + 4 \cdot 1 + 3 \cdot 2) & (1 \cdot 4 + 4 \cdot 3 + 3 \cdot 6) \\ (2 \cdot 5 + 6 \cdot 1 + 1 \cdot 2) & (2 \cdot 4 + 6 \cdot 3 + 1 \cdot 6) \\ (5 \cdot 5 + 2 \cdot 1 + 8 \cdot 2) & (5 \cdot 4 + 2 \cdot 3 + 8 \cdot 6) \end{bmatrix} = \begin{bmatrix} 15 & 34 \\ 18 & 32 \\ 43 & 74 \end{bmatrix}$$

2. The multiplication of matrices is not commutative,  $AB \neq BA$ .
3. Two vectors can be multiplied only if
  - they have the same number of elements, and
  - One is a row vector and the other is a column vector.
4. A built-in function, dot (a, b) computes the dot product of two vectors.

5. The multiplication of a column vector by a row vector, each with  $n$  elements, gives an  $n \times n$  matrix. Where the multiplication of row vector by column gives one value.

### Tutorial 3-1: Multiplication of arrays.

```
>> A=[1 4 2; 5 7 3; 9 1 6; 4 2 8]
```

```
A =
```

```
    1    4    2
    5    7    3
    9    1    6
    4    2    8
```

Define a  $4 \times 3$  matrix A.

```
>> B=[6 1; 2 5; 7 3]
```

```
B =
```

```
    6    1
    2    5
    7    3
```

Define a  $3 \times 2$  matrix B.

```
>> C=A*B
```

```
C =
```

```
    28    27
    65    49
    98    32
    84    38
```

Multiply matrix A by matrix B and assign the result to variable C.

```
>> D=B*A
```

```
??? Error using ==> *
Inner matrix dimensions must agree.
```

Trying to multiply B by A,  $B*A$ , gives an error since the number of columns in B is 2 and the number of rows in A is 4.

```
>> F=[1 3; 5 7]
```

```
F =
```

```
    1    3
    5    7
```

Define two  $2 \times 2$  matrices F and G.

```
>> G=[4 2; 1 6]
```

## Tutorial 3-1: Multiplication of arrays. (Continued)

```
G =
     4     2
     1     6
```

```
>> F*G
```

Multiply F\*G

```
ans =
     7     20
    27     52
```

```
>> G*F
```

Multiply G\*F

```
ans =
    14     26
    31     45
```

Note that the answer for G\*F is not the same as the answer for F\*G.

```
>> AV=[2 5 1]
```

Define a three-element row vector AV.

```
AV =
     2     5     1
```

```
>> BV=[3; 1; 4]
```

Define a three-element column vector BV.

```
BV =
     3
     1
     4
```

```
>> AV*BV
```

Multiply AV by BV. The answer is a scalar. (Dot product of two vectors.)

```
ans =
    15
```

```
>> BV*AV
```

Multiply BV by AV. The answer is a 3 x 3 matrix.

```
ans =
     6     15     3
     2     5     1
     8     20     4
```

```
>>
```

```
>> A=[2 5 7 0; 10 1 3 4; 6 2 11 5]
```

Define a 3 x 4 matrix A.

```
A =
     2     5     7     0
    10     1     3     4
     6     2    11     5
```

```
>> b=3
```

Assign the number 3 to the variable b.

```
b =
     3
```



```
>> b*A
```

```
ans =
```

```
    6    15    21    0
   30     3     9   12
   18     6    33   15
```

```
>> C=A*5
```

```
C =
```

```
   10    25    35    0
   50     5    15   20
   30    10    55   25
```

Multiply the matrix A by b. This can be done by either typing b\*A or A\*b.

Multiply the matrix A by 5 and assign the result to a new variable C. (Typing C = 5\*A gives the same result.)

6. Linear algebra rules of array multiplication provide a convenient way for writing a system of linear equations. For example, the system of three equations with three unknowns

$$\begin{aligned} A_{11}x_1 + A_{12}x_2 + A_{13}x_3 &= B_1 \\ A_{21}x_1 + A_{22}x_2 + A_{23}x_3 &= B_2 \\ A_{31}x_1 + A_{32}x_2 + A_{33}x_3 &= B_3 \end{aligned} \longrightarrow \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \longrightarrow$$

$$AX = B \quad \text{where } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ and } B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}.$$

## LECTURE 3-2

### 3.3 Array Division

1. The division operation is also associated with the rules of linear algebra.
2. The division operation achieved using the identity matrix and the inverse operation.

#### Identity matrix:

- ✚ The identity matrix is a square matrix in which the diagonal elements are 1 s and the rest of the elements are 0 s.
- ✚ When the identity matrix multiplies another matrix (or vector), that matrix (or vector) is unchanged. For example:

$$\begin{bmatrix} 7 & 3 & 8 \\ 4 & 11 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 8 \\ 4 & 11 & 5 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 2 \\ 15 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 15 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 6 & 2 & 9 \\ 1 & 8 & 3 \\ 7 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 9 \\ 1 & 8 & 3 \\ 7 & 4 & 5 \end{bmatrix}$$

For A square matrix

$$AI = IA = A$$

#### Inverse of a matrix:

- ✚ The matrix B is the inverse of the matrix A, both matrices must be square. We have

$$BA = AB = I$$

- ✚ The inverse of a matrix can be obtained either by  $A^{-1}$  or built-in-function  $\text{inv}(A)$ .

Not every matrix has an inverse. A matrix has an inverse only if it is square and its determinant is not equal to zero.

```
>> A=[2 1 4; 4 1 8; 2 -1 3]
```

Creating the matrix A.

```
A =
```

```
    2    1    4
    4    1    8
    2   -1    3
```

```
>> B=inv(A)
```

Use the `inv` function to find the inverse of A and assign it to B.

```
B =
```

```
    5.5000   -3.5000    2.0000
    2.0000   -1.0000    0.0000
   -3.0000    2.0000   -1.0000
```

```
>> A*B
```

Multiplication of A and B gives the identity matrix.

```
ans =
```

```
    1    0    0
    0    1    0
    0    0    1
```

```
>> A*A^-1
```

Use the power  $-1$  to find the inverse of A. Multiplying it by A gives the identity matrix.

```
ans =
```

```
    1    0    0
    0    1    0
    0    0    1
```

### Determinants:

- A determinant of the matrix is a function associated with square matrices.
- The determinant is typically denoted by  $\det(A)$  or  $|A|$ .
- For a second-order  $2 \times 2$  matrix, the rule is:

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}, \text{ for example, } \begin{vmatrix} 6 & 5 \\ 3 & 9 \end{vmatrix} = 6 \cdot 9 - 5 \cdot 3 = 39$$

### Array division:

MATLAB has two types of array division, right division and left division.

#### Left division, \:

- Left division is used to solve the matrix equation  $AX = B$  (set of linear equations).
- In this equation X and B are column vectors.
- This equation can be solved as shown below

$$A^{-1}AX = A^{-1}B \longrightarrow A^{-1}AX = IX = X \longrightarrow X = A^{-1}B \longrightarrow X = AB$$

#### Right division /:

- The right division is used to solve the matrix equation  $XC = D$ .
- In this equation X and D are row vectors.
- This equation can be solved as shown below:

$$X \cdot CC^{-1} = D \cdot C^{-1} \longrightarrow X = D \cdot C^{-1} \longrightarrow X = D/C$$

### Sample Problem 3-1:

Use matrix operations to solve the following system of linear equations.

$$4x - 2y + 6z = 8$$

$$2x + 8y + 2z = 4$$

$$6x + 10y + 3z = 0$$

### Solution

The above system of equations can be written in the matrix form  $AX = B$  or in the form  $XC = D$ :

$$\begin{bmatrix} 4 & -2 & 6 \\ 2 & 8 & 2 \\ 6 & 10 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 4 & 2 & 6 \\ -2 & 8 & 10 \\ 6 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 4 & 0 \end{bmatrix}$$

```
>> A=[4 -2 6; 2 8 2; 6 10 3];
>> B=[8; 4; 0];
>> X=A\B
X =
   -1.8049
    0.2927
    2.6341
>> Xb=inv(A)*B
Xb =
   -1.8049
    0.2927
    2.6341
```

Solving the form  $AX = B$ .

Solving by using left division:  $X = A \setminus B$ .

Solving by using the inverse of  $A$ :  $X = A^{-1}B$ .

```
>> C=[4 2 6; -2 8 10; 6 2 3];
>> D=[8 4 0];
>> Xc=D/C
Xc =
   -1.8049    0.2927    2.6341
>> Xd=D*inv(C)
Xd =
   -1.8049    0.2927    2.6341
```

Solving the form  $XC = D$ .

Solving by using right division:  $X = D/C$ .

Solving by using the inverse of  $C$ :  $X = D \cdot C^{-1}$ .

## 3.4 Element-by-Element Operations

Element-by-element operations can be done only with arrays of the same size.

Symbol	Description	Symbol	Description
.*	Multiplication	./	Right division
.^	Exponentiation	.\	Left Division

### For Vectors

If two vectors  $a$  and  $b$  are  $a = [a_1, a_2, a_3, a_4]$  and  $b = [b_1, b_2, b_3, b_4]$ , then element-by-element multiplication, division, and exponentiation of the two vectors gives:



$$a .* b = [a_1 b_1 \ a_2 b_2 \ a_3 b_3 \ a_4 b_4]$$

$$a ./ b = [a_1/b_1 \ a_2/b_2 \ a_3/b_3 \ a_4/b_4]$$

$$a.^b = [(a_1)^{b_1} \ (a_2)^{b_2} \ (a_3)^{b_3} \ (a_4)^{b_4}]$$

### For Matrices

☒ If two matrices A and B are

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \text{ and } B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

$$A .* B = \begin{bmatrix} A_{11}B_{11} & A_{12}B_{12} & A_{13}B_{13} \\ A_{21}B_{21} & A_{22}B_{22} & A_{23}B_{23} \\ A_{31}B_{31} & A_{32}B_{32} & A_{33}B_{33} \end{bmatrix} \quad A ./ B = \begin{bmatrix} A_{11}/B_{11} & A_{12}/B_{12} & A_{13}/B_{13} \\ A_{21}/B_{21} & A_{22}/B_{22} & A_{23}/B_{23} \\ A_{31}/B_{31} & A_{32}/B_{32} & A_{33}/B_{33} \end{bmatrix}$$

$$A.^n = \begin{bmatrix} (A_{11})^n & (A_{12})^n & (A_{13})^n \\ (A_{21})^n & (A_{22})^n & (A_{23})^n \\ (A_{31})^n & (A_{32})^n & (A_{33})^n \end{bmatrix}$$

**Tutorial 3-2: Element-by-element operations.**

```
>> A=[2 6 3; 5 8 4]
```

Define a 2 × 3 array A.

```
A =
```

```
     2     6     3
     5     8     4
```

```
>> B=[1 4 10; 3 2 7]
```

Define a 2 × 3 array B.

```
B =
```

```
     1     4    10
     3     2     7
```

```
>> A.*B
```

Element-by-element multiplication of array A by B.

```
ans =
```

```
     2    24    30
    15    16    28
```

```
>> C=A./B
```

Element-by-element division of array A by B. The result is assigned to variable C.

```
C =
```

```
  2.0000  1.5000  0.3000
  1.6667  4.0000  0.5714
```

## Tutorial 3-2: Element-by-element operations. (Continued)

```
>> B.^3
ans =
     1     64    1000
    27     8     343
```

Element-by-element exponentiation of array B. The result is an array in which each term is the corresponding term in B raised to the power of 3.

```
>> A*B
??? Error using ==> *
Inner matrix dimensions must agree.
```

Trying to multiply A\*B gives an error, since A and B cannot be multiplied according to linear algebra rules. (The number of columns in A is not equal to the number of rows in B.)

- ☒ Element-by-element calculations are very useful for calculating the value of a function at many values of its argument.

```
>> x=[1:8]
x =
     1     2     3     4     5     6     7     8
```

Create a vector x with eight elements.

```
>> y=x.^2-4*x
y =
    -3    -4    -3     0     5    12    21    32
>>
```

Vector x is used in element-by-element calculations of the elements of vector y.

```
>> z=[1:2:11]
z =
     1     3     5     7     9    11
>> y=(z.^3 + 5*z)./(4*z.^2 - 10)
y =
   -1.0000    1.6154    1.6667    2.0323    2.4650    2.9241
```

Create a vector z with six elements.

Vector z is used in element-by-element calculations of the elements of vector y.

```
>> x=[0:pi/6:pi]
x =
     0    0.5236    1.0472    1.5708    2.0944    2.6180    3.1416
>>y=cos(x)
y =
     1.0000    0.8660    0.5000    0.0000   -0.5000   -0.8660   -1.0000
>>
```

```
>> d=[1 4 9; 16 25 36; 49 64 81]
d =
     1     4     9
    16    25    36
    49    64    81
>> h=sqrt(d)
h =
     1     2     3
     4     5     6
     7     8     9
```

Creating a 3 × 3 array.

h is a 3 × 3 array in which each element is the square root of the corresponding element in array d.

**LECTURE 3-3****3.5 Built-in Functions for Analyzing Arrays**

MATLAB has many built-in functions for analyzing arrays. Table 3-1 lists some of these functions.

**Table 3-1: Built-in array functions**

<b>Function</b>	<b>Description</b>	<b>Example</b>
<code>mean(A)</code>	If <i>A</i> is a vector, returns the mean value of the elements of the vector.	<pre>&gt;&gt; A=[5 9 2 4]; &gt;&gt; mean(A) ans =     5</pre>
<code>C=max(A)</code>	If <i>A</i> is a vector, <i>C</i> is the largest element in <i>A</i> . If <i>A</i> is a matrix, <i>C</i> is a row vector containing the largest element of each column of <i>A</i> .	<pre>&gt;&gt; A=[5 9 2 4 11 6 11 1]; &gt;&gt; C=max(A) C =     11</pre>
<code>[d,n]=max(A)</code>	If <i>A</i> is a vector, <i>d</i> is the largest element in <i>A</i> , and <i>n</i> is the position of the element (the first if several have the max value).	<pre>&gt;&gt; [d,n]=max(A) d =     11 n =     5</pre>
<code>min(A)</code>	The same as <code>max(A)</code> , but for the smallest element.	<pre>&gt;&gt; A=[5 9 2 4]; &gt;&gt; min(A) ans =     2</pre>
<code>[d,n]=min(A)</code>	The same as <code>[d,n]=max(A)</code> , but for the smallest element.	
<code>sum(A)</code>	If <i>A</i> is a vector, returns the sum of the elements of the vector.	<pre>&gt;&gt; A=[5 9 2 4]; &gt;&gt; sum(A) ans =     20</pre>
<code>sort(A)</code>	If <i>A</i> is a vector, arranges the elements of the vector in ascending order.	<pre>&gt;&gt; A=[5 9 2 4]; &gt;&gt; sort(A) ans =     2    4    5    9</pre>
<code>median(A)</code>	If <i>A</i> is a vector, returns the median value of the elements of the vector.	<pre>&gt;&gt; A=[5 9 2 4]; &gt;&gt; median(A) ans =     4.5000</pre>



Table 3-1: Built-in array functions (Continued)

Function	Description	Example
<code>std(A)</code>	If A is a vector, returns the standard deviation of the elements of the vector.	<pre>&gt;&gt; A=[5 9 2 4]; &gt;&gt; std(A) ans =     2.9439</pre>
<code>det(A)</code>	Returns the determinant of a square matrix A.	<pre>&gt;&gt; A=[2 4; 3 5]; &gt;&gt; det(A) ans =     -2</pre>
<code>dot(a,b)</code>	Calculates the scalar (dot) product of two vectors a and b. The vectors can each be row or column vectors.	<pre>&gt;&gt; a=[1 2 3]; &gt;&gt; b=[3 4 5]; &gt;&gt; dot(a,b) ans =     26</pre>
<code>cross(a,b)</code>	Calculates the cross product of two vectors a and b, (a×b). The two vectors must have each three elements.	<pre>&gt;&gt; a=[1 3 2]; &gt;&gt; b=[2 4 1]; &gt;&gt; cross(a,b) ans =     -5     3    -2</pre>
<code>inv(A)</code>	Returns the inverse of a square matrix A.	<pre>&gt;&gt; A=[2 -2 1; 3 2 -1; 2 -3 2]; &gt;&gt; inv(A) ans =     0.2000    0.2000     0    -1.6000    0.4000    1.0000    -2.6000    0.4000    2.0000</pre>

### 3.7 Generation of Random Numbers

Simulations of many physical processes and engineering applications frequently require using a number (or a set of numbers) with a random value.

MATLAB has two commands—`rand` and `randi` can be used to assign random numbers to variables.

#### 3.7.1 The `rand` command:

The `rand` command generates uniformly distributed random numbers with values between 0 and 1.



Table 3-2: The rand command

Command	Description	Example
rand	Generates a single random number between 0 and 1.	<pre>&gt;&gt; rand ans =     0.2311</pre>
rand(1,n)	Generates an n-element row vector of random numbers between 0 and 1.	<pre>&gt;&gt; a=rand(1,4) a =     0.6068    0.4860    0.8913    0.7621</pre>
rand(n)	Generates an $n \times n$ matrix with random numbers between 0 and 1.	<pre>&gt;&gt; b=rand(3) b =     0.4565    0.4447    0.9218     0.0185    0.6154    0.7382     0.8214    0.7919    0.1763</pre>
rand(m,n)	Generates an $m \times n$ matrix with random numbers between 0 and 1.	<pre>&gt;&gt; c=rand(2,4) c =     0.4057    0.9169    0.8936    0.3529     0.9355    0.4103    0.0579    0.8132</pre>
randperm(n)	Generates a row vector with n elements that are random permutation of integers 1 through n.	<pre>&gt;&gt; randperm(8) ans =      8     2     7     4     3     6     5     1</pre>

### 3.7.2 The randi command:

The randi command generates uniformly distributed random integer. The command can be used to assign these numbers to a scalar, a vector, or a matrix, as shown in Table 3-3.

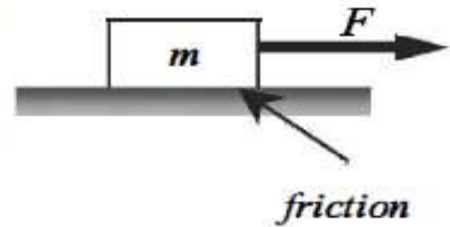
Table 3-3: The randi command

Command	Description	Example
randi(imax) (imax is an integer)	Generates a single random number between 1 and imax.	<pre>&gt;&gt; a=randi(15) a =      9</pre>
randi(imax, n)	Generates an $n \times n$ matrix with random integers between 1 and imax.	<pre>&gt;&gt; b=randi(15,3) b =      4     8    11     14     3     8      1    15     8</pre>
randi(imax, m, n)	Generates an $m \times n$ matrix with random integers between 1 and imax.	<pre>&gt;&gt; c=randi(15,2,4) c =      1     1     8    13     11     2     2    13</pre>

## Sample Problem 3-2: Friction experiment (element-by-element calculations)

The coefficient of friction,  $\mu$ , can be determined in an experiment by measuring the force  $F$  required to move a mass  $m$ . When  $F$  is measured and  $m$  is known, the coefficient of friction can be calculated by:

$$\mu = F/(mg) \quad (g = 9.81 \text{ m/s}^2).$$



Results from measuring  $F$  in six tests are given in the table below. Determine the coefficient of friction in each test, and the average from all tests.

Test	1	2	3	4	5	6
Mass $m$ (kg)	2	4	5	10	20	50
Force $F$ (N)	12.5	23.5	30	61	117	294

**Solution**

A solution using MATLAB commands in the Command Window is shown below.

```
>> m=[2 4 5 10 20 50];
>> F=[12.5 23.5 30 61 117 294];
>> mu=F./(m*9.81)
```

mu =

```
0.6371    0.5989    0.6116    0.6218    0.5963    0.5994
```

```
>> mu_ave=mean(mu)
```

mu\_ave =

```
0.6109
```

Enter the values of  $m$  in a vector.

Enter the values of  $F$  in a vector.

A value for  $\mu$  is calculated for each test, using element-by-element calculations.

The average of the elements in the vector  $\mu$  is determined by using the function `mean`.



## 3.9 Problems

- For the function  $y = x^2 - e^{0.5x} + x$ , calculate the value of  $y$  for the following values of  $x$  using element-by-element operations:  $-3, -2, -1, 0, 1, 2, 3$ .
- For the function  $y = \frac{(x+5)^3}{x^2}$ , calculate the value of  $y$  for the following values of  $x$  using element-by-element operations:  $1, 2, 3, 4, 5, 6$ .
- For the function  $y = \frac{(x+7)^4}{(x+1)\sqrt{x}}$ , calculate the value of  $y$  for the following values of  $x$  using element-by-element operations:  $1.5, 2.5, 3.5, 4.5, 5.5, 6.6$ .
- For the function  $y = \frac{2\sin x + \cos^2 x}{\sin^2 x}$ , calculate the value of  $y$  for the following values of  $x$  using element-by-element operations:  $20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ$ .
- The radius,  $r$ , of a sphere can be calculated from its surface area,  $s$ , by:

$$r = \frac{\sqrt{s/\pi}}{2}$$

The volume,  $V$ , is given by:

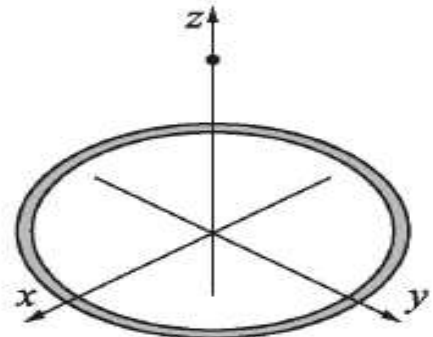
$$V = \frac{4\pi r^3}{3}$$

Determine the volume of spheres with surface area of 50, 100, 150, 200, 250, and 300 ft<sup>2</sup>. Display the results in a two-column table where the values of  $s$  and  $V$  are displayed in the first and second columns, respectively.

- The electric field intensity,  $E(z)$ , due to a ring of radius  $R$  at any point  $z$  along the axis of the ring is given by:

$$E(z) = \frac{\lambda}{2\epsilon_0} \frac{Rz}{(z^2 + R^2)^{3/2}}$$

where  $\lambda$  is the charge density,  $\epsilon_0 = 8.85 \times 10^{-12}$  is the electric constant, and  $R$  is the radius of the ring. Consider the case where  $\lambda = 1.7 \times 10^{-7}$  C/m and  $R = 6$  cm.

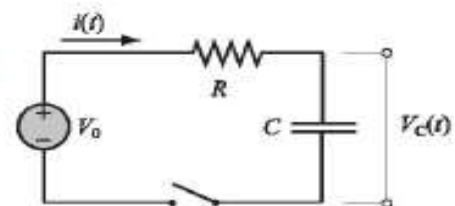


- Determine  $E(z)$  at  $z = 0, 2, 4, 6, 8,$  and  $10$  cm.
- Determine the distance  $z$  where  $E$  is maximum. Do it by creating a vector  $z$  with elements ranging from 2 cm to 6 cm and spacing of 0.01 cm. Calculate  $E$  for each value of  $z$  and then find the maximum  $E$  and associated  $z$  with MATLAB's built-in function `max`.

- The voltage  $V_C(t)$  (in V) and the current  $i(t)$  (in Amp)  $t$  seconds after closing the switch in the circuit shown are given by:

$$V_C(t) = V_0(1 - e^{-t/\tau_0})$$

$$i(t) = \frac{V_0}{R} e^{-t/\tau_0}$$





where  $\tau_0 = RC$  is the time constant. Consider the case where  $V_0 = 24$  V,  $R = 3800 \Omega$  and  $C = 4000 \times 10^{-6}$  F. Determine the voltage and the current during the first 20 s after the switch is closed. Create a vector with values of times from 0 to 20 s with spacing of 2 s, and use it for calculating  $V_C(t)$  and  $i(t)$ . Display the results in a three-column table where the values of time, voltage and current are displayed in the first, second, and third columns, respectively.

8. The length  $|\mathbf{u}|$  (magnitude) of a vector  $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  is given by  $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$ . Given the vector  $\mathbf{u} = 23.5\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ , determine its length in the following two ways:
- Define the vector in MATLAB, and then write a mathematical expression that uses the components of the vector.
  - Define the vector in MATLAB, then determine the length by writing one command that uses element-by-element operation and MATLAB built-in functions `sum` and `sqrt`.
9. A vector  $\mathbf{w}_L$  of length  $L$  in the direction of a vector  $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  can be determined by  $\mathbf{w}_L = L\mathbf{u}_n$  (multiplying a unit vector in the direction of  $\mathbf{u}$  by  $L$ ). The unit vector  $\mathbf{u}_n$  in the direction of the vector  $\mathbf{u}$  is given by  $\mathbf{u}_n = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{\sqrt{x^2 + y^2 + z^2}}$ . By writing one MATLAB command, determine a vector of length 18 in the direction of the vector  $\mathbf{u} = 7\mathbf{i} - 4\mathbf{j} - 11\mathbf{k}$ .
10. The following two vectors are defined in MATLAB:
- $$\mathbf{v} = [15, 8, -6] \quad \mathbf{u} = [3, -2, 6]$$
- By hand (pencil and paper) write what will be displayed if the following commands are executed by MATLAB. Check your answers by executing the commands with MATLAB.
- `v ./ u`
  - `u' * v`
  - `u * v'`
11. Two vectors are given:
- $$\mathbf{u} = 5\mathbf{i} - 6\mathbf{j} + 9\mathbf{k} \quad \text{and} \quad \mathbf{v} = 11\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$$
- Use MATLAB to calculate the dot product  $\mathbf{u} \cdot \mathbf{v}$  of the vectors in three ways:
- Write an expression using element-by-element calculation and the MATLAB built-in function `sum`.
  - Define  $\mathbf{u}$  as a row vector and  $\mathbf{v}$  as a column vector, and then use matrix multiplication.
  - Use the MATLAB built-in function `dot`.
12. Define the vector  $\mathbf{v} = [2 \ 3 \ 4 \ 5 \ 6]$ . Then use the vector in a mathematical expression to create the following vectors:
- $\mathbf{a} = [4 \ 6 \ 8 \ 10 \ 12]$
  - $\mathbf{b} = [8 \ 27 \ 64 \ 125 \ 216]$
  - $\mathbf{c} = [2^2 \ 3^3 \ 4^4 \ 5^5 \ 6^6]$
  - $\mathbf{d} = [1 \ 1.5 \ 2 \ 2.5 \ 3]$



13. Define the vector  $v = [8 \ 6 \ 4 \ 2]$ . Then use the vector in a mathematical expression to create the following vectors:

$$(a) \ a = [1 \ 1 \ 1 \ 1]$$

$$(b) \ b = \left[ \frac{1}{8^2} \ \frac{1}{6^2} \ \frac{1}{4^2} \ \frac{1}{2^2} \right]$$

$$(c) \ c = \left[ \frac{1}{\sqrt{8}} \ \frac{1}{\sqrt{6}} \ \frac{1}{\sqrt{4}} \ \frac{1}{\sqrt{2}} \right]$$

$$(d) \ d = [3 \ 1 \ -1 \ -3]$$

14. Define  $x$  and  $y$  as the vectors  $x = [1, 2, 3, 4, 5]$  and  $y = [2, 4, 6, 8, 10]$ . Then use them in the following expressions to calculate  $z$  using element-by-element calculations.

$$(a) \ z = \frac{(x+y)^2}{x-y}$$

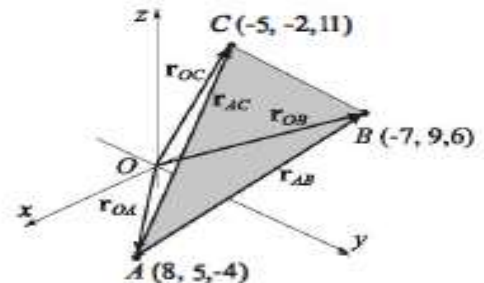
$$(b) \ w = x \ln(x^2 + y^2) + \sqrt{\frac{y^3}{(y-x)^2}}$$

15. Define  $r$  and  $s$  as scalars  $r = 1.6 \times 10^3$  and  $s = 14.2$ , and,  $t$ ,  $x$ , and  $y$  as vectors  $t = [1, 2, 3, 4, 5]$ ,  $x = [0, 2, 4, 6, 8]$ , and  $y = [3, 6, 9, 12, 15]$ . Then use these variables to calculate the following expressions using element-by-element calculations for the vectors.

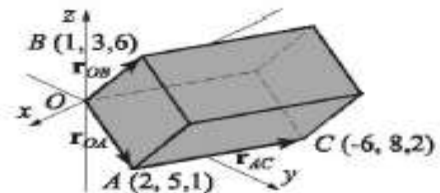
$$(a) \ G = xt + \frac{r}{s^2}(y^2 - x)t$$

$$(b) \ R = \frac{r(-xt + yt^2)}{15} - s^2(y - 0.5x^2)t$$

16. The area of a triangle  $ABC$  can be calculated by  $|\mathbf{r}_{AB} \times \mathbf{r}_{AC}|/2$ , where  $\mathbf{r}_{AB}$  and  $\mathbf{r}_{AC}$  are vectors connecting the vertices  $A$  and  $B$  and  $A$  and  $C$ , respectively. Determine the area of the triangle shown in the figure. Use the following steps in a script file to calculate the area. First, define the vectors  $\mathbf{r}_{OA}$ ,  $\mathbf{r}_{OB}$  and  $\mathbf{r}_{OC}$  from knowing the coordinates of points  $A$ ,  $B$ , and  $C$ . Then determine the vectors  $\mathbf{r}_{AB}$  and  $\mathbf{r}_{AC}$  from  $\mathbf{r}_{OA}$ ,  $\mathbf{r}_{OB}$  and  $\mathbf{r}_{OC}$ . Finally, determine the area by using MATLAB's built-in functions `cross`, `sum`, and `sqrt`.



17. The volume of the parallelepiped shown can be calculated by  $\mathbf{r}_{OB} \cdot (\mathbf{r}_{OA} \times \mathbf{r}_{OC})$ . Use the following steps in a script file to calculate the area. Define the vectors  $\mathbf{r}_{OA}$ ,  $\mathbf{r}_{AC}$ , and  $\mathbf{r}_{OB}$  from knowing position of points  $A$ ,  $B$ , and  $C$ . Determine the volume by using MATLAB's built-in functions `dot` and `cross`.



18. Define the vectors:

$$\mathbf{u} = 5\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, \quad \mathbf{v} = -2\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}, \quad \text{and} \quad \mathbf{w} = 8\mathbf{i} + 1\mathbf{j} - 3\mathbf{k}$$

Use the vectors to verify the identity:

$$(\mathbf{u} + \mathbf{v}) \cdot [(\mathbf{v} + \mathbf{w}) \times (\mathbf{w} + \mathbf{u})] = 2\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$$

Use MATLAB's built-in functions `cross` and `dot`, calculate the value of the left and right sides of the identity.

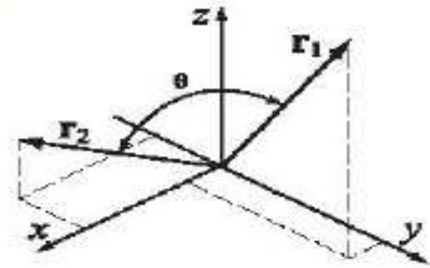


19. The dot product can be used for determining the angle between two vectors:

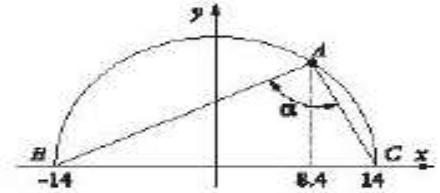
$$\theta = \cos^{-1} \left( \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1| |\mathbf{r}_2|} \right)$$

Use MATLAB's built-in functions `acosd`, `sqrt`, and `dot` to find the angle (in degrees) between  $\mathbf{r}_1 = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$  and  $\mathbf{r}_2 = 2\mathbf{i} + 9\mathbf{j} + 10\mathbf{k}$ .

Recall that  $|\mathbf{r}| = \sqrt{\mathbf{r} \cdot \mathbf{r}}$ .



20. Use MATLAB to show that the angle inscribed in a semi-circle is a right angle. Use the following steps in a script file to calculate the angle. Define a variable with the value of the  $x$  coordinate of point  $A$ . Determine the  $y$  coordinate of point  $A$  using the equation  $x^2 + y^2 = R^2$ . Define vectors that correspond to the position of points  $A$ ,  $B$ , and  $C$  and use them for determining position vectors  $\mathbf{r}_{AB}$  and  $\mathbf{r}_{AC}$ . Calculate the angle  $\alpha$  in two ways.



First by using the equation  $\alpha = \cos^{-1} \left( \frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{|\mathbf{r}_{AB}| |\mathbf{r}_{AC}|} \right)$ , and then by using the equation  $\alpha = \sin^{-1} \left( \frac{|\mathbf{r}_{AB} \times \mathbf{r}_{AC}|}{|\mathbf{r}_{AB}| |\mathbf{r}_{AC}|} \right)$ . Both should give  $90^\circ$ .

21. The position as a function of time ( $x(t)$ ,  $y(t)$ ) of a projectile fired with a speed of  $v_0$  at an angle  $\alpha$  is given by

$$x(t) = v_0 \cos \alpha \cdot t \quad y(t) = v_0 \sin \alpha \cdot t - \frac{1}{2} g t^2$$

where  $g = 9.81 \text{ m/s}^2$ . The polar coordinates of the projectile at time  $t$  are  $(r(t), \theta(t))$ , where

$$r(t) = \sqrt{x(t)^2 + y(t)^2} \quad \text{and} \quad \tan \theta(t) = \frac{y(t)}{x(t)}$$

Consider the case where  $v_0 = 162 \text{ m/s}$  and  $\alpha = 70^\circ$ . Determine  $r(t)$  and  $\theta(t)$  for  $t = 1, 6, 11, \dots, 31 \text{ s}$ .



22. Use MATLAB to show that the sum of the infinite series  $\sum_{n=0}^{\infty} \frac{2^n}{n!}$  converges to

$e^2$ . Do this by computing the sum for:

(a)  $n = 5$ , (b)  $n = 10$ , (c)  $n = 50$

For each part create a vector  $n$  in which the first element is 0, the increment is 1 and the last term is 5, 10, or 50. Then use element-by-element calculations to create a vector in which the elements are  $\frac{2^n}{n!}$ . Finally, use MATLAB's built-in function `sum` to sum the series. Compare the values to  $e^2$  (use `format long` to display the numbers).



23. Use MATLAB to show that the sum of the infinite series  $\sum_{n=1}^{\infty} \frac{(9/10)^n}{n}$  converges to  $\ln 10$ . Do this by computing the sum for (a)  $n = 10$ , (b)  $n = 50$ , (c)  $n = 100$ . For each part, create a vector  $n$  in which the first element is 1, the increment is 1 and the last term is 10, 50 or 100. Then use element-by-element calculations to create a vector in which the elements are  $\frac{(9/10)^n}{n}$ . Finally, use MATLAB's built-in function `sum` to sum the series. Compare the values to  $\ln 10$  (use `format long` to display the numbers).
24. According to Zeno's paradox any object in motion must arrive at the halfway point before it can arrive at its destination. Once arriving at the halfway point, the remaining distance is once again divided in half and so on to infinity. Since it is impossible to complete this process, Zeno concluded all motion must be an illusion. Letting the length be unity, Zeno's paradox can be written in terms of the infinite sum  $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$ . To see how quickly this series converges to 1, compute the sum for: (a)  $n = 5$ , (b)  $n = 10$ , (c)  $n = 40$ . For each part create a vector  $n$  in which the first element is 1, the increment is 1, and the last term is 5, 10, or 40. Then use element-by-element calculations to create a vector in which the elements are  $\frac{1}{2^n}$ . Finally, use the MATLAB built-in function `sum` to add the terms of the series. Compare the values obtained in parts (a), (b), and (c) with the value of 1.
25. Show that  $\lim_{x \rightarrow 0} \frac{\cos(2x) - 1}{\cos x - 1} = 4$ . Do this by first creating a vector  $x$  that has the elements 1.0, 0.5, 0.1, 0.01, 0.001, and 0.0001. Then, create a new vector  $y$  in which each element is determined from the elements of  $x$  by  $\frac{\cos(2x) - 1}{\cos x - 1}$ . Compare the elements of  $y$  with the value 4 (use `format long` to display the numbers).
26. Show that  $\lim_{x \rightarrow 1} \frac{x^{1/3} - 1}{x^{1/4} - 1} = \frac{4}{3}$ . Do this by first creating a vector  $x$  that has the elements 2.0, 1.5, 1.1, 1.01, 1.001, 1.00001, and 1.0000001. Then, create a new vector  $y$  in which each element is determined from the elements of  $x$  by  $\frac{x^{1/3} - 1}{x^{1/4} - 1}$ . Compare the elements of  $y$  with the value  $4/3$  (use `format long` to display the numbers).



27. The demand for water during a fire is often the most important factor in the design of distribution storage tanks and pumps. For communities with populations less than 200,000, the demand  $Q$  (in gallons/min) can be calculated by:

$$Q = 1020\sqrt{P}(1 - 0.01\sqrt{P})$$

where  $P$  is the population in thousands. Set up a vector for  $P$  that starts at 10 and increments by 10 up to 200. Use element-by-element computations to determine the demand  $Q$  for each population in  $P$ .

28. The ideal gas equation states that  $P = \frac{nRT}{V}$ , where  $P$  is the pressure,  $V$  is the volume,  $T$  is the temperature,  $R = 0.08206$  (L atm)/(mol K) is the gas constant, and  $n$  is the number of moles. Real gases, especially at high pressure, deviate from this behavior. Their response can be modeled with the van der Waals equation

$$P = \frac{nRT}{V - nb} - \frac{n^2a}{V^2}$$

where  $a$  and  $b$  are material constants. Consider 1 mole ( $n = 1$ ) of nitrogen gas at  $T = 300$ K. (For nitrogen gas  $a = 1.39$  (L<sup>2</sup> atm)/mol<sup>2</sup>, and  $b = 0.0391$  L/mol.) Create a vector with values of  $V$ s for  $0.1 \leq V \leq 1$  L, using increments of 0.02 L. Using this vector calculate  $P$  twice for each value of  $V$ , once using the ideal gas equation and once with the van der Waals equation. Using the two sets of values for  $P$ , calculate the percent of error ( $\frac{P_{ideal} - P_{waals}}{P_{waals}} 100$ ) for each value of  $V$ . Finally, by using MATLAB's built-in function `max`, determine the maximum error and the corresponding volume.

29. Create the following three matrices:

$$A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & 2 & 4 \\ -2 & 0 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -2 & 1 \\ 5 & 1 & -6 \\ 2 & 7 & -1 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 4 & -1 \\ 0 & 8 & 2 \\ -3 & 5 & 3 \end{bmatrix}$$

- (a) Calculate  $A + B$  and  $B + A$  to show that addition of matrices is commutative.
- (b) Calculate  $A + (B + C)$  and  $(A + B) + C$  to show that addition of matrices is associative.
- (c) Calculate  $3(A + C)$  and  $3A + 5C$  to show that, when matrices are multiplied by a scalar, the multiplication is distributive.
- (d) Calculate  $A*(B + C)$  and  $A*B + A*C$  to show that matrix multiplication is distributive.
30. Use the matrices  $A$ ,  $B$ , and  $C$  from the previous problem to answer the following:
- (a) Does  $A*B = B*A$ ? (b) Does  $A*(B*C) = (A*B)*C$ ?
- (c) Does  $(A*B)^t = A^t*B^t$ ? ( $t$  means transpose) (d) Does  $(A + B)^t = A^t + B^t$ ?



31. Create a  $4 \times 4$  matrix  $A$  having random integer values between 1 and 10. Call the matrix  $A$  and, using MATLAB, perform the following operations. For each part explain the operation.

(a)  $A * A$

(b)  $A .* A$

(c)  $A \setminus A$

(d)  $A ./ A$

(e)  $\det(A)$

(f)  $\text{inv}(A)$

32. The magic square is an arrangement of numbers in a square grid in such a way that the sum of the numbers in each row, and in each column, and in each diagonal is the same. MATLAB has a built-in function `magic(n)` that returns an  $n \times n$  magic square. In a script file create a  $(6 \times 6)$  magic square, and then test the properties of the resulting matrix by finding the sum of the elements in each row, in each column and in both diagonals. In each case, use MATLAB's built-in function `sum`. (Other functions that can be useful are `diag` and `flip1r`.)

33. Solve the following system of three linear equations:

$$-4x + 3y + z = -18.2$$

$$5x + 6y - 2z = -48.8$$

$$2x - 5y + 4.5z = 92.5$$

34. Solve the following system of five linear equations:

$$2.5a - b + 3c + 1.5d - 2e = 57.1$$

$$3a + 4b - 2c + 2.5d - e = 27.6$$

$$-4a + 3b + c - 6d + 2e = -81.2$$

$$2a + 3b + c - 2.5d + 4e = -22.2$$

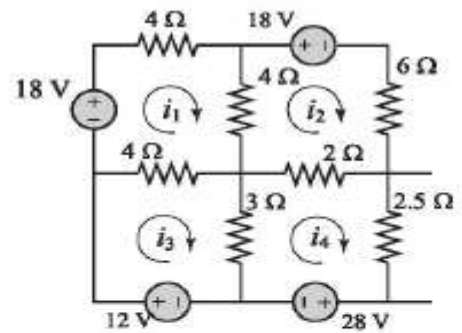
$$a + 2b + 5c - 3d + 4e = -12.2$$

35. A food company manufactures five types of 8 oz Trail mix packages using different mixtures of peanuts, almonds, walnuts, raisins, and M&Ms. The mixtures have the following compositions:

	Peanuts (oz)	Almonds (oz)	Walnuts (oz)	Raisins (oz)	M&Ms (oz)
Mix 1	3	1	1	2	1
Mix 2	1	2	1	3	1
Mix 3	1	1	0	3	3
Mix 4	2	0	3	1	2
Mix 5	1	2	3	0	2

How many packages of each mix can be manufactured if 128 lb of peanuts, 118 lb of almonds, 112 lb of walnuts, 112 lb of raisins, and 104 lb of M&Ms are available? Write a system of linear equations and solve.

36. The electrical circuit shown consists of resistors and voltage sources. Determine  $i_1, i_2, i_3,$  and  $i_4$ , using the mesh current method based on Kirchhoff's voltage law (see Sample Problem 3-4).



37. The electrical circuit shown consists of resistors and voltage sources. Determine  $i_1, i_2, i_3, i_4$  and  $i_5$ , using the mesh current method based on Kirchhoff's voltage law (see Sample Problem 3-4).

$$\begin{aligned} V_1 &= 40 \text{ V}, & V_2 &= 30 \text{ V}, & V_3 &= 36 \text{ V}, \\ R_1 &= 16 \Omega, & R_2 &= 20 \Omega, & R_3 &= 10 \Omega \\ R_4 &= 14 \Omega, & R_5 &= 8 \Omega, & R_6 &= 16 \Omega, \\ R_7 &= 10 \Omega, & R_8 &= 15 \Omega, & R_9 &= 6 \Omega, \\ R_{10} &= 4 \Omega. \end{aligned}$$

