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Anwar Kamal

Nuclear Physics

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Nuclear Physics

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ISSN 1868-4513 ISSN 1868-4521 (electronic)
Graduate Texts in Physics
ISBN 978-3-642-38654-1 ISBN 978-3-642-38655-8 (eBook)
DOI 10.1007/978-3-642-38655-8
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014941884

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Dedicated to my parents

Preface

This is an introductory textbook of nuclear physics for upper undergraduate students. The book is based on lectures given at British, American and Indian Universities over several years.

The idea of writing a text book on this subject was born some forty years ago. It is attempted to survey the major developments in nuclear physics during the past 100 years. In Rutherford's time and early 1950's, only a few Elementary particles were known and the existence of the neutrino was taken for granted. The development of the subject is so fascinating that we were inclined to present the historical facts in chronological order.

The prerequisites for the use of this book are the elements of quantum mechanics comprising Schrodinger's equation and applications, Born's approximation, the golden rule, differential equations and Vector Calculus. Basic concepts are explained with line diagrams wherever required. An attempt is made to strike a balance between theory and experiment. Theoretical predictions are compared with latest observations to show agreement or discrepancies with the theory.

The subject matter is developed in each chapter with the necessary mathematical details. Feynman diagrams are used extensively to explain the fundamental interactions. The subjects of various chapters are so much intimately connected that the logical sequential presentation of various topics became a vexing problem. For example, from the point of view of introducing quarks, the logical sequence would be strong, electromagnetic, weak and electroweak interactions, but from the point of view of introducing Feynman's diagrams, the desirable sequence would be electromagnetic, weak, electroweak and strong interactions, which is why one finds some variance in sequences for particle physics in various textbooks. The only remedy is to make cross references to the chapters which were previously studied and to those in which the relevant material is anticipated.

The size of the book did not allow to also include applied nuclear physics and cosmic rays. At the end of each chapter, a set of questions is given. A large number of worked examples is additionally presented. A comparable number of unworked problems with answers helps the student to test the understanding. The examples and problems are not necessarily of plug-in type but are given to explain the underlying physics. Useful appendices are provided at the end of the book.

Murphy, TX, USA Anwar Kamal

Note: These two volumes are the last books by my father Dr. Ahmad Kamal, the work he had conceived as his dream project and indeed his scientific masterpiece. Unfortunately, he passed away before he could see his manuscript in print. While we have tried our best to bring the publishing process to as satisfactory conclusion as possible, we regret any errors you may discover, in particular, that some of the references could not be as completely specifically cited as would otherwise be the case. We trust that these errors however do not compromise the quality or standard of the content of the text.

Suraiya Kamal
Daughter of Dr. Ahmad Kamal

Acknowledgements

I am grateful to God for helping us to complete the dream project of Dr. Ahmad Kamal after his demise, which seemed very difficult and even impossible at times.

I would like to thank Springer-Verlag, in particular Dr. Claus Asheron, Mr. Donatas Akmanavičius, Ms. Adelheid Duhm and Ms. Elke Sauer for their constant encouragement, patience, cooperation, and for bringing the book to its current form.

This project would not be complete without the constant support and encouragement of Mrs. Maryam Kamal, wife of Dr. Ahmad Kamal. Her determination kept us all moving to get these books done. My sincere thanks is due to the family, friends and well-wishers of the author who helped and prayed for the completion of these books.

Suraiya Kamal
Daughter of Dr. Ahmad Kamal

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Chapter 1
Passage of Charged Particles Through Matter

1.1 Various Types of Processes

When charged particles pass through matter, the following processes may take place:

(1) Inelastic collisions with the bound electrons of the atoms of the medium, in which case the particle energy is spent in the excitation or ionization of atoms and molecules. The energy losses of this kind of collisions are called ionization losses (collision losses) to distinguish them from radiation losses that are concerned with the generation of bremsstrahlung.
(2) Inelastic collisions with nuclei, leading to the production of bremsstrahlung quanta, to the excitation of nuclear levels, or to the nuclear reactions.
(3) Elastic collisions with nuclei, in which part of the kinetic energy of the incident particle is transferred to the recoil nuclei. However, the total kinetic energy of the colliding particles remains unchanged. A particular type of elastic scattering is the Rutherford scattering which results from the interaction of a charged particle with the Coulomb field of the target nucleus in single encounters. When thick materials are used, cumulative single scatterings give rise to the phenomenon of multiple scattering.
(4) Elastic collisions with bound electrons.
(5) Cerenkov effect, i.e. emission of light by charged particles passing through matter with a velocity exceeding the velocity of light waves in the given medium.

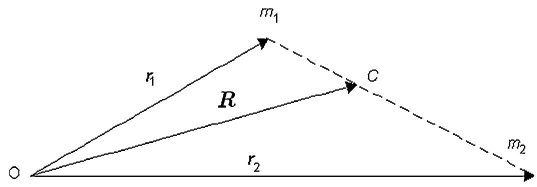
1.2 Kinematics

1.2.1 Laboratory (Lab) System (LS) and Centre of Mass System (CM)

In order to describe the motion of particles in the collision problem one must choose a definite frame of reference (co-ordinate system). Two frames of reference are im-

A. Kamal, *Nuclear Physics*, Graduate Texts in Physics,
DOI 10.1007/978-3-642-38655-8_1,

Fig. 1.1 The position vectors m_1 and m_2 and their centre of mass is shown



portant, one is the lab system (LS) and the other one is centre of mass system (CMS). In the lab system, the observer who is at rest in the lab views the collision process. In the CM system the centre of mass is at rest initially and always. Observations are usually made in the lab system but theoretical calculations are made in the CM system. It is of great interest to find out how various quantities like velocity, angle of scattering, etc. are related in these two systems. It is easier to perform calculations in the CM system rather than in the lab system. For, the great merit of CM system is that the total linear momentum of particles is always zero so that in the two-body process particles move directly towards each other before the collision and they recede in the opposite direction after the collision.

The collision process in the CM system may be visualized as the one in which a particle of reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ moving with initial velocity u_1 collides with a fixed scattering centre. Here, u_1 is the initial velocity of m_1 moving towards the target particle of mass m_2 at rest.

1.2.2 Total Linear Momentum in the CM System Is Zero

In Fig. 1.1, the position of the centre of mass of two particles m_1 and m_2 is shown by C. The position of masses m_1 and m_2 are indicated by the position vectors r_1 and r_2 and that of the centre of mass by R. By definition

$$\boldsymbol{R} = \frac{m_1 r_1 + m_2 r_2}{M} \quad \text{or}$$

$$M\boldsymbol{R} = m_1 r_1 + m_2 r_2$$

Differentiating with respect to time

$$M\dot{R} = m_1 \dot{r} + m_2 \dot{r} \quad \text{or}$$

$$M v_c = m_1 u_1 + m_2 u_2$$

where u_1 and u_2 are the initial velocities of particles 1 and 2, respectively and v_c is the CM velocity. Since m_2 is initially at rest, $u_2 = 0$, and the centre of mass which is located at $M = m_1 + m_2$, must move in the lab system towards m_2 with velocity

$$v_c = \frac{m_1 u_1}{m_1 + m_2} \tag{1.1}$$