Coding Theory

Sheet 7

Spring 2014

1. In a binary linear code, show that either all or precisely half the words have even weight.

2. Let C be a binary $[n,k]$ code. Given $W_{C^\perp}(T)$, find an expression for $W_C(T)$.

3. Let C be the binary $[6,3]$ code with parity-check matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

   (a) Find a generator matrix G for C.

   (b) Find the weight enumerator $W_C(T)$ by writing out all the elements of C.

   (c) Apply MacWilliams' theorem to $W_C(T)$ to obtain the weight enumerator $W_{C^\perp}(T)$.

   (d) Find the weight enumerator $W_{C^\perp}(T)$ by writing out all the elements of $C^\perp$.

   (e) What phenomenon do you observe?

4. Let C be the binary $[10,7]$ code with parity-check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

   Find the weight enumerator $W_C(T)$ by first finding $W_{C^\perp}(T)$ and then applying the MacWilliams' theorem. Hence, write down the weight distribution of C.

5. (a) For $C = \mathrm{Ham}(r,2)$, use the result of Sheet 6, Exercise 7 to write down $W_{C^\perp}(T)$.

   (b) Deduce $W_C(T)$.

   (c) Use the same technique to do the previous parts of this question for $C = \mathrm{Ham}(r,q)$, but here with the homogeneous weight enumerator $\overline{W}_C(X,Y)$. *(Harder)*

6. Let C be the $[6,2]_7$ code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 4 & 2 & 3 & 6 \\ 0 & 1 & 4 & 6 & 5 & 2 \end{bmatrix}.$$

   (a) If the rows of G are x and y, find the weights of the eight codewords y, $x + ty$ for $t \in \mathbf{F}_7$.

   (b) Deduce the weight distribution of C.

   (c) Write down $\overline{W}_C(X,Y)$.

   (d) Calculate $\overline{W}_{C^\perp}(X,Y)$.