Coding Theory

Sheet 5

Spring 2014

1. An $[n, k]$ code C is *self-dual* if $C = C^\perp$ and *weakly self-dual* if $C \subset C^\perp$.

   (a) If C is self-dual, what is k?

   (b) Construct both a binary, weakly self-dual code C that is not self-dual and its dual $C^\perp$ when $n = 4$, with $C \cap C^\perp \neq \{0\}$.

2. Let G be the generator matrix of the binary $[5, 3]$ code C given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

   (a) Write out a standard array for C. (b) Find a parity-check matrix for C.

3. Let G be the generator matrix of the ternary $[3, 2]$ code C given by

$$G = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}.$$

   (a) Write out a standard array for C. (b) Find a parity-check matrix for C.

4. For the code
$$C_3 = \{00000, 01101, 10110, 11011\},$$

   (a) write out a standard array;

   (b) use the array to correct the messages (i) 10011; (ii) 10111.

5. Let C be the binary code with generator matrix

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

(a) Find the number of codewords in C.

(b) Find a parity-check matrix for C.

(c) Find the minimum distance of C.

(d) Find the number of errors that C can (i) detect, (ii) correct.

6. Let H be a parity-check matrix for the binary $[n, k, d]$ code C, where

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Find $n, k, d, |C|$.

(b) Find a generator matrix for C. What is $d^\perp = d(C^\perp)$?

(c) Show that $C^\perp \subset C$.

(d) Find coset leaders and their syndromes.

(e) Decode the following received vectors: (i) 1110101; (ii) 1110011.

7. Let G be a generator matrix for the binary code $[n, k, d]$ code C, where

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

(a) Find a parity-check matrix for C.

(b) Find n, k, $|C|$, $d = d(C)$, $d^\perp = d(C^\perp)$.

(c) Find coset leaders and their syndromes.

(d) Decode the following received vectors: (i) 100010; (ii) 111100.