Coding Theory

Sheet 4

Spring 2014

1. What is the dimension of the subspace spanned by

   (a) 1011, 1111, 1001, 1101 in $V(4, 2)$;

   (b) 1210, 1021, 1011, 0212 in $V(4, 3)$?

2. * Show that the number of k-dimensional linear codes of $V(n, q)$ is

$$\frac{(q^n - 1)(q^{n-1} - 1) \cdots (q^{n-k+1} - 1)}{(q^k - 1)(q^{k-1} - 1) \cdots (q - 1)}.$$

   (Hint: Choose a basis.)

3. Find a generator matrix for the perfect $[7, 4, 3]$ code C derived from the projective plane of order 2 and then find a generator matrix for C in standard form.

4. * Let C be the binary $[6, 3]$ code with generator matrix

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

   Find a generator matrix for C in standard form only using row operations.

5. Let C be the ternary $[7, 4]$ code with generator matrix

$$G = \begin{bmatrix} 2 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 2 & 0 \\ 2 & 1 & 0 & 2 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}.$$

   Find a generator matrix for C in standard form only using row operations.

6. * Let C be the $[5, 4]$ code over $\mathbf{F}_7$ with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 3 & 5 & 4 \\ 0 & 0 & 2 & 3 & 5 \\ 2 & 1 & 0 & 3 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

   Find a generator matrix for C in standard form only using row operations.

7. Let $G = [I_k\, A]$ be the generator matrix of a code C and let $G' = [I_k\, A']$, where the rows of A' are a permutation of the rows of A. Show that G' is the generator matrix of a code C' equivalent to C.

8. Let C be a binary code of length n. Form a binary code C' of length $n+1$ as follows:

$$x = x_1 x_2 \cdots x_n \in C \Longrightarrow x' = x_1 x_2 \cdots x_n x_{n+1} \in C',$$

where

$$x_{n+1} = \begin{cases} 1 & \text{if } w(x) \text{ is odd,} \\ 0 & \text{if } w(x) \text{ is even.} \end{cases}$$

Show that, if C is linear, then C' is also linear; it is called the *extended code.*

9. Assign characters to elements of $V(4,2)$ as follows:

space	→	0000	D	→	1000	M	→	1001	S	→	1110
A	→	0001	E	→	0011	N	→	1010	T	→	1011
B	→	0010	F	→	0110	O	→	0101	U	→	1101
C	→	0100	R	→	0111	G	→	1100	Y	→	1111

Encode these information 4−tuples in a $[7,4]_2$ code using the generator matrix

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Show that the corresponding code C is single-error correcting and decode the following received sequences:

(a) 1110001 1100011 1001100 0100111 0110000 1000011 0001011;

(b) 1010110 1000011 0000011 0001000 1101111 0000101 1110101 0000000 0011110 0100101 1100101 1011010;

(c) 0011000 1100001 0000110 0111011 1010011 0001101 0010111 0001000 1010110 1110011 1110101 1010000 0000011 1111010 0010011.

Do it by inspection; there is no need to write out a standard array.

10. For the code C in Question 4,

(a) write out a standard array;

(b) use the array to correct the messages (i) 011101; (ii) 001011.

As part of the course assessment, hand in at the School Office solutions to the starred questions, namely 2, 4, 6, by 2.00 p.m. on Thursday, 20th February. Solutions to all questions will be placed online on Friday, 21st February.