Coding Theory

Sheet 2

Spring 2014

1. Using the argument of the Sphere Packing Bound, state an upper bound for M for a q-ary (n, M, d) code when $d = 2e + 2$, that is, d is even.

2. * Use the Sphere Packing Bound (Theorem 2.3) to find an upper bound for M of a binary $(5, M, 3)$ code.

3. * The previous exercise gives an upper bound for $A_2(5, 3)$. Now, show by construction that $A_2(5, 3) = 4$.

4. Show, by construction, that $A_2(8, 5) = 4$.

5. What is the covering radius of each of the codes $C_1,\ C_2,\ C_3$?

6. Show that the binary repetition code of length n, with n odd, is perfect. How many errors does it correct?

7. * What are the packing and covering radii of the ternary repetition code of length 6.

8. Show that the perfect $(7, 16, 3)$ code C derived from the projective plane of order 2 is *linear*; that is, the sum modulo 2 of any two elements of C is in C.

9. Show that a perfect binary $(n, M, 7)$ code has $n = 7$ or $n = 23$.

   (*Harder*!)

10. Find the multiplicative inverses of all non-zero elements in the following fields:

    (a) $\mathbf{F}_5$; (b) $\mathbf{F}_7$; (c)* $\mathbf{F}_{13}$; (d) $\mathbf{F}_{17}$.

11. Solve the pair of equations $2x + y = 1, x + 2y = 1$ when the field is each of the four cases of the previous question.

As part of the course assessment, hand in at the School Office solutions to the starred questions, namely 2, 3, 7, 10(c), by 2 p.m. on Thursday, 6th February. Solutions to all questions will be placed online on Friday, 7th February.