Coding Theory

Sheet 1

Spring 2014

1. Consider the code
$$C = \{11000, 01101, 10110, 00011\}.$$
   (a) Find the distance between every pair of codewords.
   (b) What is the minimum distance $d(C)$?
   (c) Decode the following received words using nearest neighbour decoding:
   (i) 01111; (ii) 10110; (iii) 11011; (iv) 10011.

2. Let C be the ternary repetition code of length 4 over the alphabet $\{0, 1, 2\}$.
   (a) List the vectors which will be uniquely decoded as 1111 using nearest neighbour decoding.
   (b) If the probability of each symbol being wrongly received is t and each symbol is equally likely, find the word error probability; that is, the probability P_e of a word being incorrectly decoded.
   (c) What is P_e when $t = 0.05$?

3. Let C be the binary repetition code of length 5. List the vectors which will be uniquely decoded as 11111 using nearest neighbour decoding. Do parts (b) and (c) of the previous question for this code.

4. (The direct sum construction) Given $x = x_1 \cdots x_n$ and $y = y_1 \cdots y_m$, let
$$(x \mid y) = x_1 \cdots x_n \, y_1 \cdots y_m.$$
If C_1 is an (n, M_1, d_1) code and C_2 is an (m, M_2, d_2) code, let
$$C_3 = \{(x \mid y) \mid x \in C_1, y \in C_2\}.$$
Show that C_3 is an $(n + m, M_1 M_2, d)$ code. What is d?

5. Prove Lemma 2.5: A ball of radius r in $(\mathbf{F}_q)^n$ has size
$$|S(x, r)| = \binom{n}{0} + \binom{n}{1}(q - 1) + \cdots + \binom{n}{r}(q - 1)^r.$$