Coding Theory

Sheet 5 Solutions

Spring and Summer 2010

1. With

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

the following table is calculated.

Symbol	$\rightarrow$	u	$\rightarrow$	uG
space	$\rightarrow$	0000	$\rightarrow$	0000000
A	$\rightarrow$	0001	$\rightarrow$	1101001
B	$\rightarrow$	0010	$\rightarrow$	0101010
C	$\rightarrow$	0100	$\rightarrow$	1001100
D	$\rightarrow$	1000	$\rightarrow$	1110000
E	$\rightarrow$	0011	$\rightarrow$	1000011
F	$\rightarrow$	0110	$\rightarrow$	1100110
G	$\rightarrow$	1100	$\rightarrow$	0111100
M	$\rightarrow$	1001	$\rightarrow$	0011001
N	$\rightarrow$	1010	$\rightarrow$	1011010
O	$\rightarrow$	0101	$\rightarrow$	0100101
R	$\rightarrow$	0111	$\rightarrow$	0001111
S	$\rightarrow$	1110	$\rightarrow$	0010110
T	$\rightarrow$	1011	$\rightarrow$	0110011
U	$\rightarrow$	1101	$\rightarrow$	1010101
Y	$\rightarrow$	1111	$\rightarrow$	1111111

As the minimum weight of a codeword is 3, so $d(C) = 3$. Hence C is single-error correcting. The decoded messages are as follows:

(a) DECODER

(b) SEE YOU SOON

(c) MASTERS STUDENT

2. (a) If $C = C^\perp$, then $n - k = k$; so $k = \frac{1}{2}n$.

   (b) Let $C = \{0000, 1111\}$. Then C is a $[4,1]_2$ code and $C^\perp$ is a $[4,3]_2$ code; $C^\perp = \{0000, 1111, 0011, 1100, 0101, 1010, 0110, 1001\}$.

3. (a) As $|V(5,2)| = 32$, $|C| = 8$, there are four cosets.

C	00000	10011	01011	00101	11000	10110	01110	11101
$10000 + C$	10000	00011	11011	10101	01000	00110	11110	01101
$00100 + C$	00100	10111	01111	00001	11100	10010	01010	11001
$00010 + C$	00010	10001	01001	00111	11010	10100	01100	11111

   (b) If $G = [I_3\ A]$, then a parity-check matrix is $H = [A^T\ I_2]$. Hence

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

4. (a) As $|V(3,3)| = 27$, $|C| = 9$, there are three cosets.

C	000	210	120	012	021	222	201	102	111
$100 + C$	100	010	220	112	121	022	001	202	211
$200 + C$	200	110	020	212	221	122	101	002	011

   (b) By row operations,

$$G = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

   Hence $H = [1\ 1\ 1]$.