Curve Fitting with Functions Other than Polynomials

* Many situations in science and engineering require fitting functions that are not polynomials to given data.
* For a particular data set, however, some functions provide a better fit than others.
* In addition, determining the best-fitting coefficients can be more difficult for some functions than for others.
* This section covers curve fitting with power, exponential, logarithmic, and reciprocal functions, which are commonly used. The forms of these functions are:



* All of these functions can easily be fitted to given data with the polyfit function.
* The logarithmic function is already in this form, and the power, exponential, and reciprocal equations can be rewritten as:



These equations describe a linear relationship

* between ln(y) and ln(x) for the power function,
* between ln(y) and x for the exponential function,
* between y and ln(x) or log(x) for the logarithmic function, and
* between l/y and x for the reciprocal function.
* This means that the polyfit (x,y,l) function can be used to determine the best-fit constants m and b for best fit if, instead of x and y, the following arguments are used.



* The result of the polyfit function is assigned to p, which is a two-element vector.
* The first element, p(1), is the constant m,
* the second element, p ( 2 ) , is
* ***b for the logarithmic and reciprocal functions***,
* ***ln(b) or log(b) for the exponential function, and***
* ***ln(b) for the power function***
* ***( b = ep(2) or b = 10P(2) for the exponential function, and b = ep(2) for the power function).***
* For given data it is possible to estimate, to some extent, which of the functions has the potential for providing a good fit. This is done by plotting the data using different combinations of linear and logarithmic axes. If the data points in one of the plots appear to fit a straight line, the corresponding function can provide a good fit according to the list below.



• Exponential functions cannot pass through the origin.

• Exponential functions can fit only data with all positive y's or all negative y's.

• Logarithmic functions cannot model x= 0 or negative values of x.

• For the power function y= 0 when x=0.

• The reciprocal equation cannot model y=0.

Other considerations in choosing a function:

***Ex / Fitting an equation to data points***

Determine a function w = f(t) that best fits the data.



**Solution**

1. The data is first plotted with linear scales on both axes. The figure indicates that a linear function will not give the best fit since the points do not appear to line up along a straight line.



2. From the other possible functions, the logarithmic function is excluded since for the first point t = 0, and the power function is excluded since at t = 0, w ≠ 0.

3. To check if the other two functions (exponential and reciprocal) might give a better fit, two additional plots, shown below, are made. The plot on the left has a log scale on the vertical axis and linear horizontal axis. In the plot on the right, both axes have linear scales, and the quantity 1/w is plotted on the vertical axis.



4. In the left figure, the data points appear to line up along a straight line. This indicates that an exponential function of the form y=bemx can give a good fit to the data. A program in a script file that determines the constants b and m, and that plots the data points and the function is given below.









***Modification to polyfit function***

* In addition to the power, exponential, logarithmic, and reciprocal functions, many other functions can be written in a form suitable for curve fitting with the polyfit function.
* A function of the form

 $y=e^{a\_{2}x^{2 }+a\_{1 }x^{1}+a\_{0}}$

 is fitted to data points using the polyfit function with a

 second-order polynomial is described in example below.

***Example 2: Temperature dependence of viscosity***

Viscosity, **μ** , is a property of gases and fluids that characterizes their resistance to flow. For most materials, viscosity is highly sensitive to temperature. Below is a table that gives the viscosity of SAE 10 W oil at different temperatures.

Determine an equation that can be fitted to the data.



Solution

To determine what type of equation might provide a good fit to the data, **μ** is plotted as a function of T with a linear scale for T and a logarithmic scale for **μ**· The plot, shown below indicates that the data points do not appear to line up along a straight line. This means that a simple exponential function of the form y = bemx, which models a straight line with these axes, will not provide the best fit. Since the points in the figure appear to lie along a curved line, a function that can possibly have a good fit to the data is:

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This function can be fitted to the data by using MATLAB's polyfit (x, y, 2) function, where the independent variable is T and the dependent variable is ln(**μ**).



The following program determines the best fit to the function and creates a plot that displays the data points and the function.



When the program executes, the coefficients that are determined by the polyfit function are displayed in the Command Window (shown below) as three elements of the vector p.



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p = 0.0003 -0.2685 47.1673

With these coefficients the viscosity of the oil as a function of temperature is:



The plot that is generated shows that the equation correlates well to the data points.



Problems

1.



2.

3.

4.

5.

