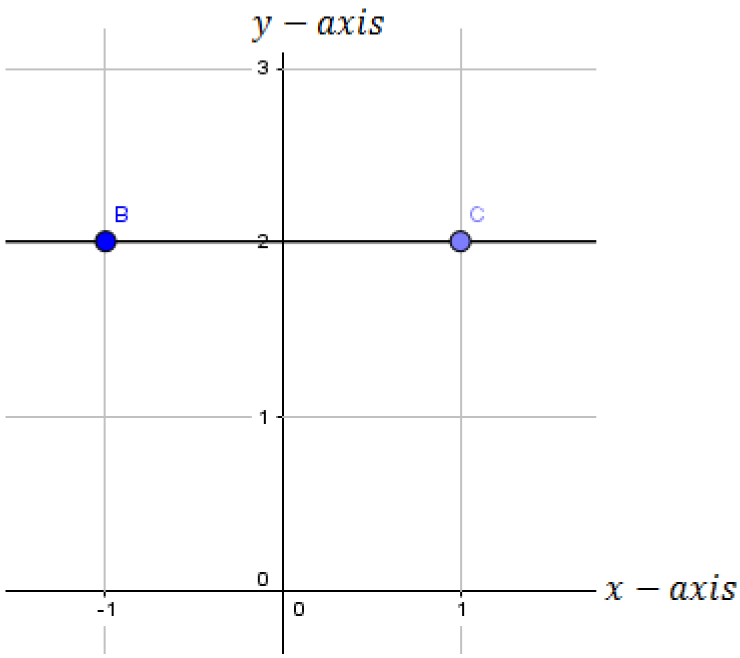
Examples 1.2.2.

(i)(Constant Function). $f: \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 2, \forall x \in \mathbb{R}$. $D(f) = \mathbb{R}$, $R(f) = \{1\}$, $Cod(f) = \mathbb{R}$.

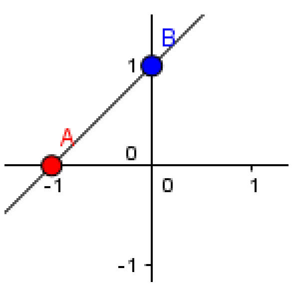


(ii) (Restriction Function). $f: \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x + 1, \forall x \in \mathbb{R}$.

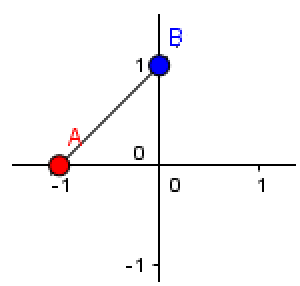
$D(f) = \mathbb{R}$, $R(f) = \mathbb{R}$, $Cod(f) = \mathbb{R}$. Let $A = [-1,0]$.

$g = f|_A: A \longrightarrow \mathbb{R}$. $g(x) = f(x) = x + 1, \forall x \in A$.

$D(g) = A$, $R(f) = [-1,1]$, $Cod(f) = \mathbb{R}$.



$f(x) = x + 1$



$g = f|_A$

(iii) (Extension Function). $f: [-1,0] \longrightarrow \mathbb{R}$, $f(x) = x + 1, \forall x \in [-1,0]$.

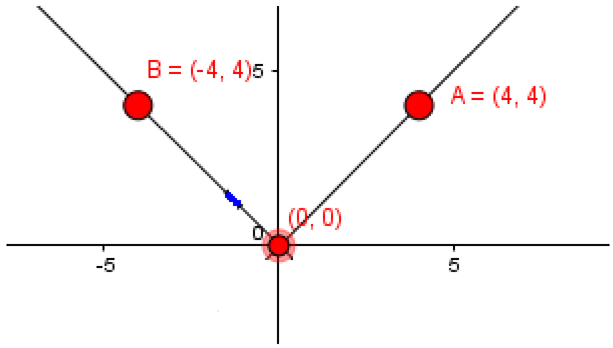
$D(f) = [-1,0]$, $R(f) = [-1,1]$, $Cod(f) = \mathbb{R}$.

Let $A = \mathbb{R}$. $g: A \longrightarrow \mathbb{R}$. $g(x) = f(x) = x + 1, \forall x \in A$.

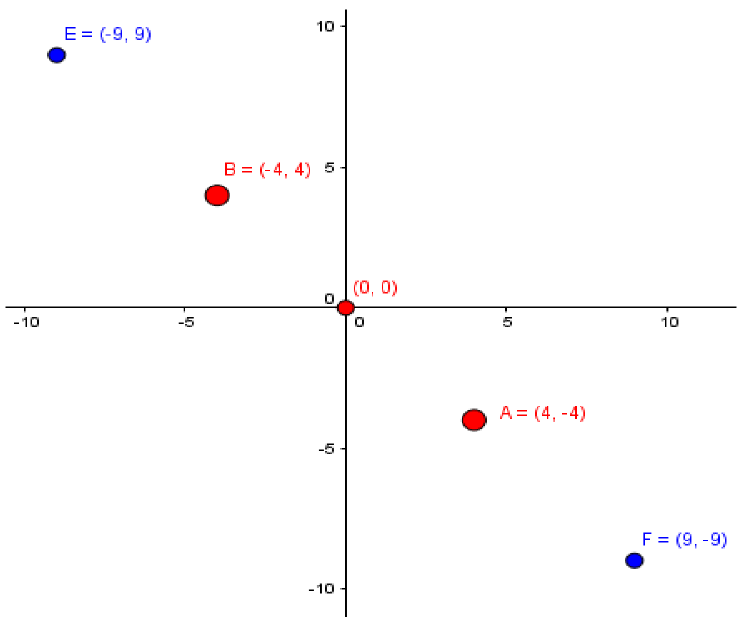
$D(g) = A,\ R(g) = \mathbb{R}, Cod(g) = \mathbb{R}$.

(iv) (Absolute Value Function) $f: \mathbb{R} \longrightarrow \mathbb{R},\ f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x & x < 0 \end{cases}$.

$D(f) = \mathbb{R}, , R(f) = [0, \infty), Cod(f) = \mathbb{R}$.



(v) (Permutation Function). $f: \mathbb{N} \longrightarrow \mathbb{N}, f(x) = -x, \forall x \in \mathbb{N}$. The function is bijective, so it is permutation function. $D(f) = \mathbb{N}, , R(f) = \mathbb{N}, Cod(f) = \mathbb{N}$.



(vi) (Sequence). $f: \mathbb{N} \longrightarrow \mathbb{Q}, f(n) = \frac{1}{n}, \forall x \in \mathbb{N}.\ \{f_n\} = \{\frac{1}{n}\}_{n=1}^{\infty}$.

(vii) (Canonical Function). Let R be an equivalence relation defined on $\mathbb{Z}$ as follows:

$$xRy \text{ iff } x-y \text{ is even integer, that is, } R=\{(x,y)\in\mathbb{Z}\times\mathbb{Z}: x-y \text{ even}\}.$$

$[0]=\{x\in\mathbb{Z}: x-0 \text{ even}\}=\{\dots,-4,-2,0,2,4,\dots\}=[2]=[-2]=\cdots.$

$[1]=\{x\in\mathbb{Z}: x-1 \text{ even}\}=\{\dots,-5,-3,-1,1,3,5,\dots\}=[-1]=[3]=\cdots.$

$\mathbb{Z}/R=\{[0],[1]\}.$

$\pi(0)=[0]=\pi(2)=\pi(-2)=\cdots.$

$\pi(1)=[1]=\pi(-1)=\pi(-3)=\cdots.$

(viii) (Projection Function)

$P_1:\mathbb{Z}\times\mathbb{Q}\longrightarrow\mathbb{Z}$, $P_1(x,y)=x$ for all $(x,y)\in\mathbb{Z}\times\mathbb{Q}$. $P_1\left(2,\frac{2}{5}\right)=2$. $P_1(\mathbb{Z},\frac{2}{5})=\mathbb{Z}.$

${P_1}^{-1}(3)=\{3\}\times\mathbb{Q}.$

(ix) (Cross Product of Functions)

$f:\mathbb{N}\longrightarrow\mathbb{Q}$, $f(n)=\frac{1}{n}, \forall n\in\mathbb{N}$ and $f:\mathbb{N}\longrightarrow\mathbb{N}$, $f(x)=-x, \forall x\in\mathbb{N}$

$$f\times g:\mathbb{N}\times\mathbb{N}\longrightarrow\mathbb{Q}\times\mathbb{N},\ (f\times g)(x,y)=(f(x),g(y))$$

$$=(\tfrac{1}{x},-y) \text{ for all } (x,y)\in\mathbb{N}\times\mathbb{N}.$$

Exercise 1.2.3.

(i) Let R be an equivalence relation defined on $\mathbb{N}$ as follows:

$$R=\{(x,y)\in\mathbb{N}\times\mathbb{N}: x-y \text{ divisble by } 3\}.$$

1- Find $\mathbb{N}/R$. **2-** Find $\pi([0]), \pi([1]), \pi^{-1}([2])$.

(ii) Prove that the Projection function is onto but not injective.

(iii) Prove that the Identity function is bijective.

(iv) Prove that the inclusion function is bijective.

(v) Let $f: A_1 \longrightarrow A_2$ and $g: B_1 \longrightarrow B_2$ be two functions. If f and g are both 1-1 (onto), then, $f \times g$ is 1-1(onto).

(vi) If $f: X \longrightarrow Y$ is a bijective function, then f^{-1} is bijective function.

(vii) If $f: X \longrightarrow Y$ is a bijective function, then

1- $f \circ f^{-1} = I_Y$ is bijective function. **2-** $f^{-1} \circ f = I_X$ is bijective function.

(viii) Let $f: X \longrightarrow Y$ and If $g: Y \longrightarrow X$ are functions. If $g \circ f = I_X$, then f is injective and g is onto.

(ix) Let $f: \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ be a function defined as follows:

$$f(x, y) = x^2 + y^2.$$

1- Find the $f(\mathbb{R} \times \mathbb{R})$ (image of f).

2- Find $f^{-1}([0,1])$.

3- Does f 1-1 or onto?

4- Let $A = \{(x, y) \in \mathbb{R} \times \mathbb{R}: x = \sqrt{2 - y^2}\}$. Find $f(A)$.