1.1 Measure of Space and Time: Units1 and Dimensions

 We shall assume that space and time are described strictly in the Newtonian sense. Three-dimensional space is Euclidian, and positions of points in that space are specified by a set of three numbers (x, y, z) relative to the origin (0,0,0) of a rectangular Cartesian coordinate system.

A length is the spatial separation of two points relative to some standard length.

Time is measured relative to the duration of reoccurrences of a given configuration of a cyclical system—say, a pendulum swinging back and forth, an Earth rotating about its axis, or electromagnetic waves from a cesium atom vibrating inside a metallic cavity. The time of occurrence of any event is specified by a number t, which represents the number of reoccurrences of a given configuration of a chosen cyclical standard. For example, if 1 vibration of a standard physical pendulum is used to define 1 s, then to say that some event occurred at t = 2.3 s means that the standard pendulum executed 2.3 vibrations after its "start" at t =0, when the event occurred. All this sounds simple enough, but a substantial difficulty has been swept under the rug: Just what are the standard units? The choice of standards has usually been made more for political reasons than for scientific ones. For example, to say that a person is 6 feet tall is to say that the distance between the top of his head and the bottom of his foot is six times the length of something, which is taken to be the standard unit of 1 foot.

The second, the basic unit of time in SI, began as an arbitrary fraction (1/86,400) of a mean solar day (24 x 60 x 60= 86,400). The trouble with astronomical clocks, though, is that they do not remain constant. The mean solar day is lengthening, and the lunar month, or time between consecutive full phases, is shortening. In 1956 a new second was defined to be 1/31,556,926 of one particular and carefully measured mean solar year, that

of 1900. That second would not last for long! In 1967 it was redefined again, in terms of a specified number of oscillations of a cesium atomic clock.

A cesium atomic dock consists of a beam of cesium-133 atoms moving through an evacuated metal cavity and absorbing and emitting microwaves of a characteristic resonant frequency, 9,192,631,770 Hertz (Hz), or about 1010 cycles per second. This absorption and emission process occurs when a given cesium atom changes its atomic configuration and, in the process, either gains or loses a specific amount of energy in the form of microwave

radiation. The two differing energy configurations correspond to situations in which the spins of the cesium nucleus and that of its single outer-shell electron are either opposed (lowest energy state) or aligned (highest energy state). This kind of a "spin-flip" atomic transition is called a hyperfine transition. The energy difference and, hence, the resonant frequency are precisely determined by the invariable structure of the cesium atom. It does

not differ from one atom to another. A properly adjusted and maintained cesium clock can keep time with a stability of about 1 part m 10. Thus, in one year, its deviation from the right time should be no more than about 30 Ps (30 x s).

The units (kilogram, meter, and second) constitute the basis of the SI system. Other systems are commonly used also, for example, the cgs (centimeter, gram, second) and the fps (foot, pound, second) systems. Regard these systems as secondary because they defined relative to the SI standard.

1.2 Vectors

The motion of dynamical systems is described in terms of two basic quantities: scalars and vectors. A scalar is a physical quantity that has magnitude only, such as the mass of an object. It is completely specified by a single number, in appropriate units. Its value is independent of any coordinates chosen to describe the motion of the system. Other familiar examples of scalars include density, volume, temperature, and energy. Mathematically, scalars are treated as real numbers. They obey all the normal algebraic rules of addition, subtraction, multiplication, division, and so on.

A vector, however, has both magnitude and direction, such as the displacement from one point in space to another. Unlike a scalar, a vector requires a set of numbers for its complete specification. The values of those numbers are, in general, coordinate system dependent. Besides displacement in space, other examples of vectors include velocity, acceleration, and force. Mathematically, vectors combine with each other according to the parallelogram rule of addition. The vector concept has led to the emergence of a branch of mathematics that has proved indispensable to the development of the subject of classical mechanics. Vectors provide a compact and elegant

way of describing the behavior of even the most complicated physical systems. A vector, for example, A. In this text, however, for the sake of simplicity, we denote vector quantities simply by boldface type, for example, A. We use ordinary italic type to represent scalars, for example, A.

A given vector A is specified by stating its magnitude and its direction relative to some arbitrarily chosen coordinate system. It is represented diagrammatically as a directed line segment, as shown in three-dimensional space in Figure 1.3.1.

A vector can also be specified as the set of its components, or projections onto the coordinate axes. For example, the set of three scalars, (Ax, Ay), shown in Figure 1.1, are the components of the vector A and are an equivalent representation. Thus, the equation

 

Implies that either the symbol **A** or the set of three components (Ar, referred to a particular coordinate system can be used to specified the vector. For example, if the vector **A** represents a displacement from a point P1 (x1, Yi' z1) to the point P2 (x2, Y2' z2), then its three components are

, ,

and the equivalent representation of A is its set of three scalar components, (, ). If A represents a force, then is the x-component of the force, and so on.

If a particular discussion is limited to vectors in a plane, only two components are necessary for their specification. In general, one can define a mathematical space of any number of dimensions n. Thus, the set of n-numbers (A1, A2, A3 A0) represent a vector in an n-dimensional space. In this abstract sense, a vector is an ordered set of numbers.

We begin the study of vector algebra with some formal statements concerning vectors.

**I. Equality of Vectors**

The equation A=B or =

is equivalent to the three equations

 , ,

That is, two vectors are equal if, and only if, their respective components are equal. Geometrically, equal vectors are parallel and have the same length, but they do not necessarily have the same position. Equal vectors are shown in Figure 1.3.2. Though equal, they are physically separate. (Equal vectors are not necessarily equivalent in all respects. Thus, two vectorially equal forces acting at points on an object may produce different mechanical effects.)



II. Vector Addition

The addition of two vectors is defined by the equation **A+B=**

or  **+**  =

The sum of two vectors is a vector whose components are sums of the components of the given vectors. The geometric representation of the vector sum of two nonparallel vectors is the third side of a triangle, two sides of which are the given vectors. The vector sum is illustrated in Figure 1.3.3. The sum is also given by the parallelogram rule, as shown in the figure. The vector sum is defined, however, according to the above equation even if the vectors do not have a common point.



III. Multiplication by a Scalar: If **c** is a scalar and A is a vector,

The product cA is a vector whose components are c times those of A. Geometrically, the vector cA is parallel to A and is c times the length of A. When c = -1, the vector -A is one whose direction is the reverse of that of A, as shown in Figure 13.4.



**IV Vector Subtraction**

Subtraction is defined as follows:

 A-B= A+(-1) B or

or  **-**  =

That is, subtraction of a given vector B from the vector A is equivalent to adding -B to A.

**V. The Null Vector**

The vector 0 = (0,0,0) is called the null vector. The direction of the null vector is undefined. From (IV) it follows that A - A =0. Because there can be no confusion when the null vector is denoted by a zero, we shall hereafter use the notation 0=0.

**VI. The Commutative Law of Addition**

This law holds for vectors; that is,

A+B=B+A (1.3.6)

because = and similarly for the y and z components.

**VII. The Associative Law**

The associative law is also true, because

A+(B+C) = (()

=+ = (A + B) + C

**VIII. The Distributive Law**

Under multiplication by a scalar, the distributive law is valid because, from (II) and (III),

 c(**A + B**) = c

 =

 =

 =

**IX. Magnitude of a Vector**

The magnitude of a vector A, denoted by IAI or byA, is defined as the square root of the sum of the squares of the components, namely,

where the positive root is understood. Geometrically, the magnitude of a vector is its length, that is, the length of the diagonal of the rectangular parallelepiped whose sides are and expressed in appropriate units. See Figure 1.3.5



**X. Unit Coordinate Vectors**

A unit vector is a vector whose magnitude is unity. Unit vectors are often designated by the symbol e, from the German word Einheit. The three unit vectors

 , ,

are called unit coordinate vectors or basis vectors. In terms of basis vectors, any vector can be expressed as a vector sum of components as follows:

 = +

 =

 = + +

A widely used notation for Cartesian unit vectors uses the letters i, j, and k, namely, ,

We shall usually employ this notation hereafter.

The directions of the Cartesian unit vectors are defined by the orthogonal coordinate axes, as shown in Figure 1.3.6. They form a right-handed or a left-handed





