Vector and Vector space

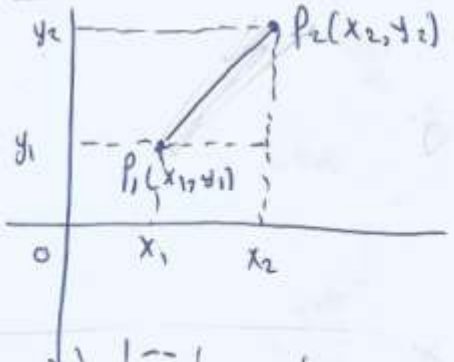
هنالك تطبيقات في الحياة تتعامل مع قيم
او (magnitude) مثل السرعة
والقوة والكتلة وهنالك تطبيقات تحتاج الى
قيم واتجاه سميت vector .

The length or magnitude of the vector

$X = (x, y)$ is

$$\|X\| = \sqrt{x^2 + y^2}$$

the length of directed Line segment with intial point $P_1(x_1, y_1)$ and terminal point $P_2(x_2, y_2)$



المسافة بين نقطتين في vector

$P_2(x_2, y_2)$ $P_1(x_1, y_1)$

$$\|\overrightarrow{P_1P_2}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

قانون المسافة بين النقطتين P_1 , P_2

ايجاد الزاوية بين متجهين بالقانون

$$\cos\theta = \frac{X \cdot Y}{\|X\| \|Y\|} \qquad 0 \le \theta \le \pi$$

<u>Def</u> $\boxed{\text{inner product}}$ or $\boxed{\text{dot product}}$ of the vectors $X = (x_1, y_1)$ and $Y = (x_2, y_2)$ is

$$X \cdot Y = x_1 x_2 + y_1 y_2$$

<u>Example</u>: If $X = (2,4)$ and $Y = (-1,2)$ then

$$X \cdot Y = (2)(-1) + (4)(2) = 6$$

and $\|X\| = \sqrt{(2)^2 + (4)^2} = \sqrt{20}$

and $\|Y\| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

$$\cos\theta = \frac{6}{\sqrt{20} \cdot \sqrt{5}} = 0.6$$

we Can obtain the approximate angle by using a table of Cosines.

orthogonal or perpendicular

If X and Y are at right angles then the Cosine of the angle θ between X and Y is zero

then from

$$\cos\theta = \frac{X \cdot Y}{\|X\|\|Y\|}$$

$$\Rightarrow \quad X \cdot Y = 0 \quad \Longrightarrow$$

X and Y is orthogonal $\Longleftrightarrow$

$$X \cdot Y = 0$$

<u>Ex</u> The Vectors $X = (2,-4)$, $Y = (4,2)$ are orthogonal since

$$X \cdot Y = (2)(4) + (-4)(2) = 0$$

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inner product / dot product من صفات

(1) $X \cdot X = \|X\|^2 \geq 0$

Unit Vectors :—

A unit vector is vector whose length is 1.

If X is any non zero, then the vector

$U = \frac{1}{\|X\|} \cdot X$ is a unit vector in the direction of X.

Ex let $X = (-3, 4)$ Then

$\|X\| = \sqrt{(-3)^2 + 4^2} = 5$

Hence the vector $U = \frac{1}{5}(-3, 4) = (-\frac{3}{5}, \frac{4}{5})$ is a unit vector since

$$\|U\| = \sqrt{\left(-\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9+16}{25}} = 1$$

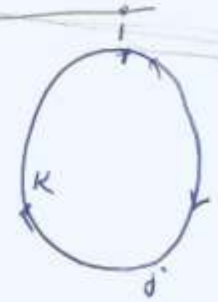
ملاحظة The set of all <u>n-vectors</u> is denoted by $\underline{R}^n$ and is called $\boxed{n\text{-space.}}$

the norm of n vector is

$$\|X\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

Def $\boxed{\text{cross product in } R^3}$

$$X \times Y = \begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$



$$X \times Y = \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} i - \begin{vmatrix} x_1 & x_3 \\ y_1 & y_3 \end{vmatrix} j + \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} k$$

مثال :- اذا كان المتجهين U , V حيث

$U = -3i - 2j + 2k$

$V = -2i + 2j + 3k$

:- الضرب الاتجاهي $U \times V$ يساوي

$$U \times V = \begin{vmatrix} i & j & k \\ -3 & -2 & 2 \\ -2 & 2 & 3 \end{vmatrix}$$

$$= i\underbrace{(-2\times3 - (2\times2))}_{-10} - j\underbrace{(-3\times3 - (2\times(-2)))}_{+5} + k\underbrace{(-3\times2 - (-2\times-2))}_{-10}$$

$$= -10i + 5j - 10k$$

<u>vector space</u>: is a set of element V together with two operations $\oplus$ and $\odot$ satisfying the following properties.

(1) If X and Y any elements of V then
$X \oplus Y$ is in V (V is closed under the $\oplus$)
(a) $X \oplus Y = Y \oplus X$ (b) $X \oplus (Y \oplus Z) = (X \oplus Y) \oplus Z$
(c) there is a unique element 0 in V such that
$$X \oplus 0 = 0 \oplus X = X$$
(d) For each X in V $\exists!$ element $-X \in V$ s.t
$$X \oplus -X = 0$$
(2) If X and $Y \in V \Rightarrow X \odot Y \in V$
(a) $c \odot (X \oplus Y) = (c \odot X) \oplus (c \odot Y)$
(b) $(c+d) \odot X = (c \odot X) \oplus d \odot X$
(c). $\boxed{1 \odot X = X}$

The element of V called vector
The real number are called scalars.
The operation $\oplus$ is called vector addition
The operation $\odot$ is called scalar multiplication
The vector 0 is called zero vector
The vector $-X$ is called the negative of X