Q1

IF the surface tempareture T_o is $283°K$, and the Surface pressure p_o is 1000hpa, Calculate the pressure and the tempareture at height of 5 Km?

To calculate the pressure we used the equation

$$P = P_o \, e^{-\frac{gZ}{R T_o}}$$

$Z = 5 \text{ Km} = 5000 \text{ m}$

$$P = 1000 \, e^{-\frac{9.8 \times 5000}{287 \times 283}}$$

$$P = 547 \text{ hpa.}$$

to calculate the tempareture we used the equation

$$T = T_o - \frac{gZ}{R}$$

$$T = 283 - \frac{9.8 \times 5000}{287}$$

$$T = 112°K.$$

Q/ integrate the H.E to find an expression for the temparature as a function of height, what is the temparature at the Middle of the atmosphere?

from H.E $\frac{dP}{dz} = -\rho g$ $\rho = \rho_0$

$dP = -\rho_0 g\, dz$ $\int_{P_0}^{P} dP = -\rho_0 g \int_{z_0}^{z} dz$

$P\Big|_{P_0}^{P} = -\rho_0 g z$ $P - P_0 = -\rho_0 g z$

$P = P_0 - \rho_0 g z$ —— ①

from equation of state $P = \rho_0 R T$

$T = \frac{P}{\rho_0 R}$ —— ② put ① in ②

$T = \frac{P_0 - \rho_0 g z}{\rho_0 R} = \underbrace{\frac{P_0}{\rho_0 R}}_{T_0} - \frac{\cancel{\rho_0} g z}{\cancel{\rho_0} R}$

$\boxed{T = T_0 - \frac{gz}{R}}$

at the middle of the atmosphere

$z = \frac{8000}{2} = 4000\,m$

$T = T_0 - \frac{gz}{R} \Rightarrow T = 273 - \frac{9.8 \times 4 \times 10^3}{287} = \boxed{136.4^\circ K}$

Q1

intgrate the hydrostatic equation to find an expression of the pressure as a function of height?

$$T = T_0 = \text{constant}$$

$$\frac{dp}{dz} = -\rho g \qquad \rho = \frac{p}{RT_0}$$

$$\frac{dp}{dz} = -\frac{gp}{RT_0} \Longrightarrow \frac{dp}{p} = -\frac{g}{RT_0}dz$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{RT_0}\int_{z_0}^{z} dz$$

$$\ln\frac{p}{p_0} = -\frac{g}{RT_0}z$$

$$\frac{p}{p_0} = e^{-\frac{gz}{RT_0}}$$

$$\therefore p = p_0 e^{-\frac{gz}{RT_0}}$$

Q1/ Show that " The scale height for an isothermal is equal to the height of the homogenous atmosphere".

To prove that $Hs = H$

$$p = p_o e^{-\frac{gz}{RT_o}} \quad \text{——} ① \text{ ---- isothermal}$$

$$p = p_o e^{-1} \quad \text{——} ②$$

$$\cancel{p_o} \cancel{e}^{-\frac{gz}{RT_o}} = \cancel{p_o} \cancel{e}^{-1}$$

$$\frac{gz}{RT_o} = 1 \quad \text{where } z = Hs$$

$$Hs = \frac{RT_o}{g} = \frac{287 \times 273}{9.8} = 8000\,m$$

$$\frac{dp}{dz} = -\rho_o g \Rightarrow \int_{p_o}^{p} dp = -\rho_o g \int_{z_o}^{z} dz \text{ --- homogenous}$$

$$p - p_o = -\rho_o g z \quad \text{when } p = 0 \quad z = H$$

$$0 = p_o - \rho_o g H \Rightarrow H = \frac{p_o}{\rho_o g} \longrightarrow p_o = \rho_o R T_o$$

$$H = \frac{\cancel{\rho_o} R T_o}{\cancel{\rho_o} g} = \frac{RT_o}{g} = \frac{287 \times 273}{9.8} = 8000\,m$$

$$\therefore Hs = H = 8\,km.$$

3 - The Atmosphere with constant laps rate.

The laps rate is $\gamma = -\dfrac{dT}{dz}$

$$\int_{T_0}^{T} dT = -\gamma \int_{z_0}^{z} dz \Rightarrow T - T_0 = -\gamma z$$

$$\boxed{\therefore\ T = T_0 - \gamma z} \quad \text{—— (1)}$$

From hydrostat equation $\dfrac{dp}{dz} = -\rho g$

$P = \rho R T \qquad \rho = \dfrac{P}{RT}$

$$\boxed{\frac{dp}{dz} = -\frac{gP}{RT}} \quad \text{—— (2)}$$

put (1) in (2)

$$\frac{dp}{dz} = -\frac{gP}{R(T_0 - \gamma z)}$$

$$\frac{dp}{p} = -\frac{g}{R(T_0 - \gamma z)}\, dz$$

let us $u = T_0 - \gamma z \qquad du = 0 - \gamma dz$

$$dz = -\frac{du}{\gamma}$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{R} \int_{0}^{z} \frac{-\frac{du}{\gamma}}{u}$$

$$\ln \frac{p}{p_0} = +\frac{g}{R\gamma} \int_{0}^{z} \frac{du}{u}$$

$$\ln \frac{p}{p_0} = +\frac{g}{R\gamma} \ln (u) \Big|_{0}^{z} \qquad u = T_0 - \gamma z$$

$$\ln \frac{p}{p_0} = \frac{g}{R\gamma} \ln (T_0 - \gamma z) \Big|_{0}^{z}$$

$$\ln \frac{p}{p_0} = \frac{g}{R\gamma} \left[\ln (T_0 - \gamma z) - \ln (T_0) \right]$$

$$\ln \frac{p}{p_0} = \frac{g}{R\gamma} \ln (T_0 - \gamma z) - \frac{g}{R\gamma} \ln (T_0).$$

where $T = T_0 - \gamma z$

$$\ln \frac{p}{p_0} = \frac{g}{R\gamma} \ln \frac{T}{T_0}$$

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$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{g}{R\gamma}} \Rightarrow p = p_0 \left(\frac{T}{T_0}\right)^{\frac{g}{R\gamma}}$$

$$p = \rho R T \qquad p_0 = \rho_0 R T_0$$

$$\rho_z R T_z = \rho_0 R T_0 \left(\frac{T}{T_0}\right)^{\frac{g}{R\gamma}}$$

$$\rho \cancel{R} T = \rho_0 \cancel{R} T_0 \left(\frac{T}{T_0}\right)^{\frac{g}{R\gamma}}$$

$$\rho = \rho_0 \left(\frac{T}{T_0}\right)^{-1} \left(\frac{T}{T_0}\right)^{\frac{g}{R\gamma}}$$

$$\boxed{\therefore \rho = \rho_0 \left(\frac{T}{T_0}\right)^{\frac{g}{R\gamma} - 1}}$$

4- The Adibatic Atmosphere:

$$\theta = \theta_0 = \text{constant.}$$

$$\theta = T\left(\frac{p}{p_1}\right)^{\frac{R}{c_p}}$$ using chane rule.

$$\cancelto{0}{\frac{1}{\theta}\frac{\partial \theta}{\partial z}} = \frac{1}{T}\frac{\partial T}{\partial z} - \frac{R}{c_p}\frac{1}{p}\frac{\partial p}{\partial z} = 0$$

$$\frac{\partial p}{\partial z} = -\rho g \quad , \quad p = \rho R T \Rightarrow \frac{R}{p} = +\frac{1}{\rho T}$$

$$\frac{1}{T}\frac{\partial T}{\partial z} = \frac{R}{c_p}\frac{1}{p}\frac{\partial p}{\partial z}$$

$$\frac{1}{T}\frac{\partial T}{\partial z} = -\frac{g}{c_p T}$$

$$\frac{1}{T}\frac{\partial T}{\partial z} + \frac{g}{c_p T} = 0$$

$$\frac{\partial T}{\partial z} = -\frac{g}{c_p}$$ $g = 9.8$, $c_p = 1004$

$$\therefore \frac{\partial T}{\partial z} = -\frac{9.8}{1004} = 10\,{}^{\circ}K/Km.$$