The Governing Equations. المعادلات الحاكمة

The equations that govern the atmosphere on Synoptic Scale are:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + fv \quad \text{--- (1)}$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial y} - fu \quad \text{--- (2)}$$

$$\frac{\partial P}{\partial z} = -\rho g \quad \text{--- (3)}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho \vec{V}) = 0 \quad \text{--- (4)}$$

$$dH = c_v \frac{dT}{dt} + P\frac{d\alpha}{dt} \quad \text{--- (5)}$$

$$P = \rho R T \quad \text{--- (6)}$$

we have six equations with six unknows (u, v, w, P, T and ρ).

Theoretically, they can be solved to predict or diagnose the future value of the 6 Variables.

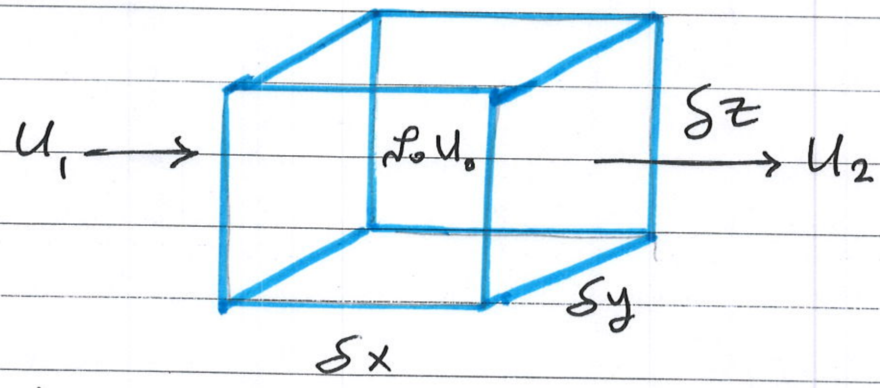
3 - The continuity Equation.

The Continuity equation comes from the law of conservation of mass, " there is no sources or sinks of mass anywhere in the atmosphere.

Consider the box at a fixed point in space. The net change in mass is found by adding up the mass fluxes entering and leaving through each face of the box.

the volume of box is $\delta V = \delta x \delta y \delta z$

The mass equal $m = \rho \delta V$.



The change of mass per unit time is:

$$\frac{\partial m}{\partial t} = \frac{\partial \rho}{\partial t} \delta V.$$

The net inflow in the x-direction is:

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$$\rho_1 u_1 \delta y \delta z - \rho_2 u_2 \delta y \delta z = 0$$

$$\frac{\partial \rho}{\partial t}\delta v = \left[\rho_0 u_0 - \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right]\delta y\,\delta z - \left[\rho_0 u_0 + \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right]\delta y\,\delta z .$$

$$\frac{\partial \rho}{\partial t}\delta v = -\frac{\partial(\rho u)}{\partial x}\delta v \quad \text{--- } x$$

Similarly the net inflow in y and z direction are:

$$\frac{\partial \rho}{\partial t}\delta v = -\frac{\partial(\rho v)}{\partial y}\delta v . \quad \text{--- } y$$

$$\frac{\partial \rho}{\partial t}\delta v = -\frac{\partial(\rho w)}{\partial z}\delta v \quad \text{--- } z$$

the Total change of mass per unit time will be:

$$\frac{\partial \rho}{\partial t}\cancel{\delta v} = -\left[\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right]\cancel{\delta v}$$

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}\cdot(\rho\vec{V})$$

$$\boxed{\therefore \frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V}) = 0}$$ The first forms.

$$\text{or} \quad \frac{\partial \rho}{\partial t} + \rho \vec{\nabla} \cdot \vec{V} + \vec{V} \cdot \vec{\nabla} \rho = 0$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} + \rho \vec{\nabla} \cdot \vec{V} = 0$$

$$\boxed{\therefore \frac{d\rho}{dt} + \rho (\vec{\nabla} \cdot \vec{V}) = 0}$$ The second forms.

The physical meaning of continuity equation (first form) is that:

The Change in density at a fixed point in space is dependent upon the divergence of the mass flux.

* If there is divergence of the mass flux then $\vec{\nabla} \cdot \rho \vec{V} > 0$ and density will decrease.
* If there is convergence of mass flux then $\vec{\nabla} \cdot \rho \vec{V} < 0$ and density will be increase.

$\vec{\nabla} \cdot (\rho \vec{V}) > 0$ اذا كان هناك تباعد في فيض الكتلة في عنصر كالهواء فإن الكثافة سوف تتناقص.

$\vec{\nabla} \cdot (\rho \vec{V}) < 0$ اذا كانت هناك تقارب في فيض الكتلة في عنصر كالهواء الكثافة سوف تتزايد.